Lines, Lines, Lines!!!
Parallel & Perpendicular Lines
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Lesson Plan

I. Topic: Parallel & Perpendicular Lines

II. Goals and Objectives:
A. The students will identify parallel lines as having the same slopes.
B. The students will identify perpendicular lines as having reciprocal slopes.
C. The students will find the parallel or perpendicular form of a given set of information.

III. Massachusetts Learning Standards:

1. 8.P.3
Demonstrate an understanding of the identity (-x)(-y) = xy. Using this identity to simplify algebraic expressions.

2. 8.P.5
Identify the slope of a line as a measure of its steepness and as a constant rate of change from its table of values, equation, or graph. Apply the concept of slope to the solution of problems.

3. 8.P.6
Identify the roles of variables within an equation, e.g., y = mx + b, expressing y as a function of x with parameters m and b.

4. 8.P.7
Set up and solve linear equations and inequalities with one or two variables, using algebraic methods, models, and/or graph.

5. 10.G.8
Find linear equations that represent lines either perpendicular or parallel to a given line and through a point, e.g., by using the "point-slope" form of the equation.

6. 10.P.2
Demonstrate an understanding of the relationship between various representations of a line. Determine a line's slope and x- and y-intercepts from its graph or from a linear equation that represents the line. Find a linear equation describing a line from a graph or geometric description of the line, e.g, by using the "point-slope" or "slope y-intercept" formulas. Explain the significance of a positive, negative, zero, or undefined slope.

7. 10.P.8
Solve everyday problems that can be modeled using systems of linear equations or inequalities. Apply algebraic and graphical methods to the solution. Use technology when appropriate. Include mixture, rate, and work problems.

IV. Materials:
A. Whiteboard with dry-erase markers (Blackboard with Chalk could also be used.)
B. Ruler.
C. Pencils.
D. Calculator.
E. Graphing Paper.
F. Parallel & Perpendicular Lines Worksheets (Practice Worksheet, Quiz Worksheet).

V. Presentation Outline:
A. Introduction. "Understanding the differences and similarities between parallel and perpendicular lines."
I. Definitions
II. Review of Slopes

B. Determine if two lines are parallel or perpendicular.
Example

C. Write the equation of a line given one point and a parallel line.
Example

D. Write the equation of a line give one point and a perpendicular line.
Examples

VI. Presentation:
A. Presentation Notes
B. Power Point Presentation

VII. Independent Practice: Parallel & Perpendicular Lines Worksheet
A. Class work: # 1 - 8 all

B. Homework: # 9 - 16 all

C. Due the next day. Allow students to complete those questions which they were not able to complete in class.

VIII. Topic Assessment: Parallel Lines Quiz
A. Answer questions from homework.

B. 5-Question Quiz: 10 – 15 minutes

C. 10-Question Quiz: 15 – 20 minutes.

Lines, Lines, Lines!!!

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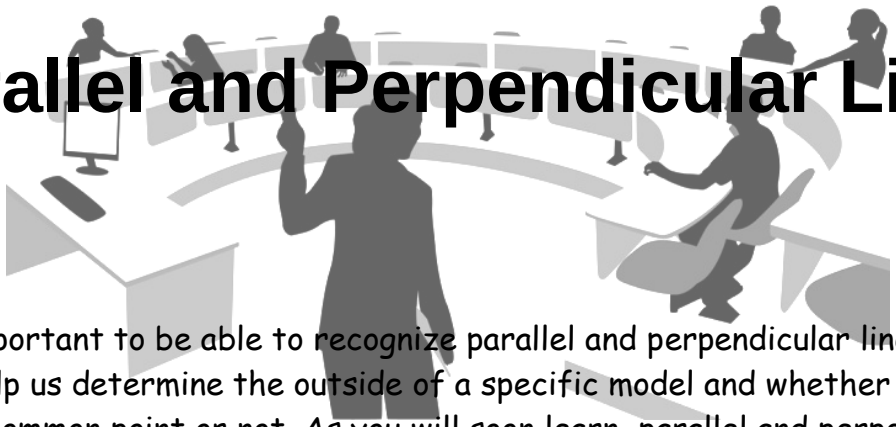
Parallel and Perpendicular Lines



Introduction

It is important to be able to recognize parallel and perpendicular lines because they help us determine the outside of a specific model and whether situations will have a common point or not. As you will soon learn, parallel and perpendicular lines have specific characteristics which help us easily identify them.

Definitions and Formula

Lines which lie on the same plane which do not intercept are said to be Parallel Lines.

Parallel Lines:

- Go indefinitely without touching (Do not intercept).
- Have the same slope.
- Vertical Lines are Parallel Lines, to each other.

Two lines that intercept each other at exactly a right angle are said to be Perpendicular lines.

Perpendicular Lines:

- Form a right angle.
- Slopes are opposite reciprocals of each other
- Vertical Lines are Perpendicular to Horizontal Lines.

Parallel and Perpendicular lines can be found in any of the previous equations we have learned.

A. $y = mx + b$ ← Slope-Intercept Form

B. $y - y_1 = m(x - x_1)$ ← Point-Slope Form

C. $Ax + By = C$ ← Standard Form

D. $y = b$ ← Horizontal Line (Zero Slope)

E. $x = b$ ← Vertical Line (Undefined Slope)

When two or more equations are placed on one graph, they are said to be part of a Family of graph. Understanding the lines and their equations helps us determine which family they belong too.

Remember:

(x, y) ← x and y represent all of the points on the line. Often referred to as (x_1, y_1) and (x_2, y_2).

'm' (lower case m) ← is the symbol used to represent the slope.

$$m = \frac{Change\ of\ y\ or\ \Delta y}{Change\ of\ x\ or\ \Delta x} = \frac{Rise}{Run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{+\uparrow\ -\downarrow}{+\rightarrow\ -\leftarrow}$$

'b' (lower case b) ← is the symbol used to represent the y-intercept. We always look for this number on the y-axis to begin our graph. The letter 'm' and the letter 'b' are constants. In other words, these are numbers which never change. They are constants.

Key Concepts

It is important to learn how to recognize when an equation is in the slope-intercept form and when it is not.

The following sets of lines are parallel lines. (same slope):

A. 1. $y = -\frac{2}{3}x + 6$

2. $y = -\frac{2}{3}x - 2$

B. 1. $y = 5x - 7$

2. $y = 5x + \frac{2}{5}$

C. 1. $y = \frac{6}{7}x$

2. $y = \frac{6}{7}x + 12$

D. 1. $y = \frac{9}{7}x - 17$

2. $y = \frac{9}{7}x - 1$

The following sets of lines are Perpendicular lines (Reciprocal slope, exact opposite slopes):

A. 1. $y = \frac{4}{3}x + 5$

2. $y = -\frac{3}{4}x$

B. 1. $y = 4x - 2$

2. $y = -\frac{1}{4}x + 7$

C. 1. $y = -\frac{2}{3}x + 6$

2. $y = \frac{3}{2}x + 1$

D. 1. $y = -3x + 10$

2. $y = \frac{1}{3}x - 5$

The following sets of lines are neither Parallel nor Perpendicular lines.

A. 1. $y = -\frac{2}{3}x + 9$

2. $y = x - 2$

B. 1. $y = -5x - 7$

2. $y = 5x - 7$

C. 1. $y = \frac{6}{7}x$

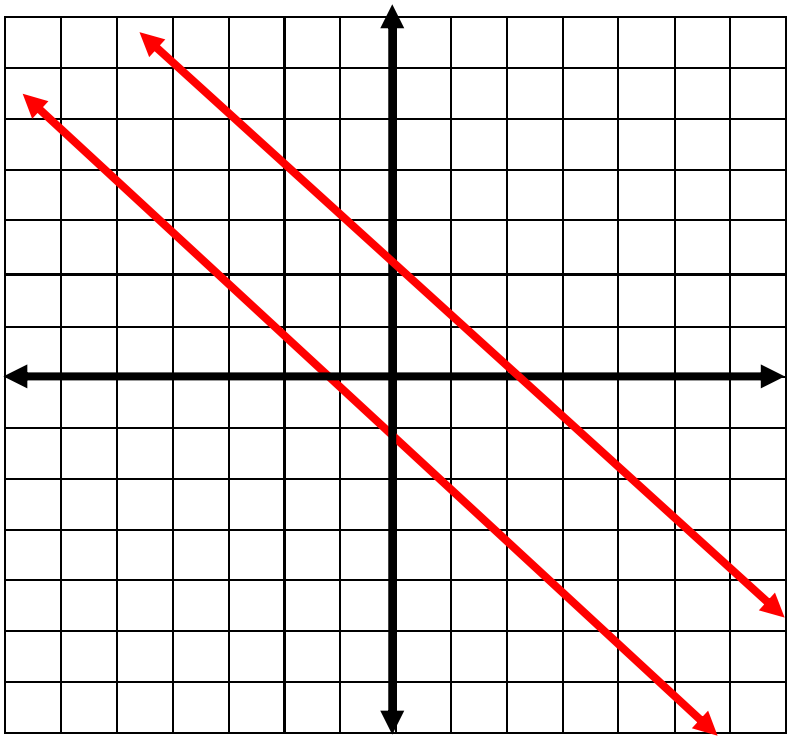
2. $x = \frac{6}{7}y$

D. 1. $y = -\frac{9}{7}x - 17$

2. $y = -\frac{7}{9}x - 17$

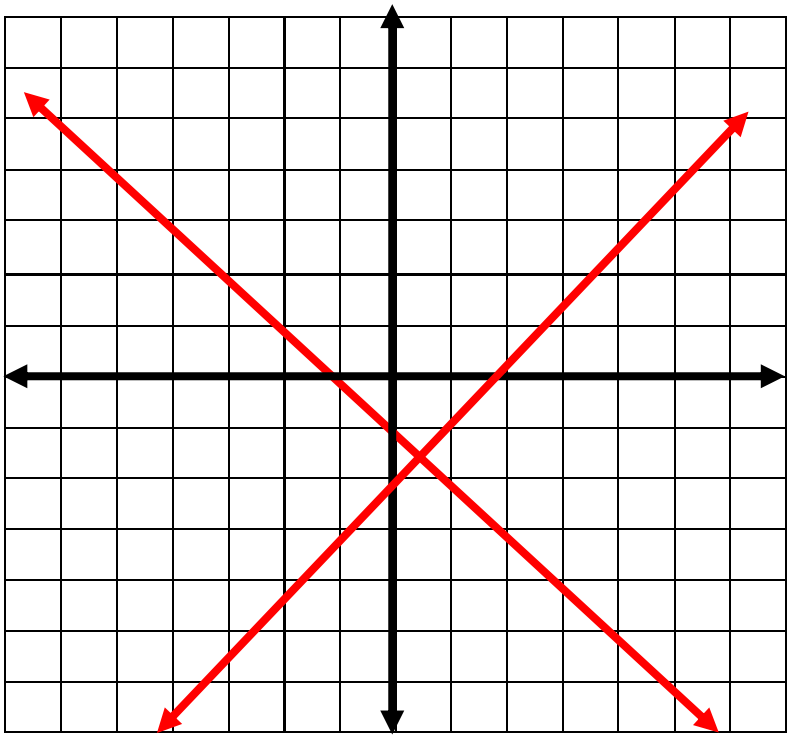
Picture this!

Two parallel lines never intersect.



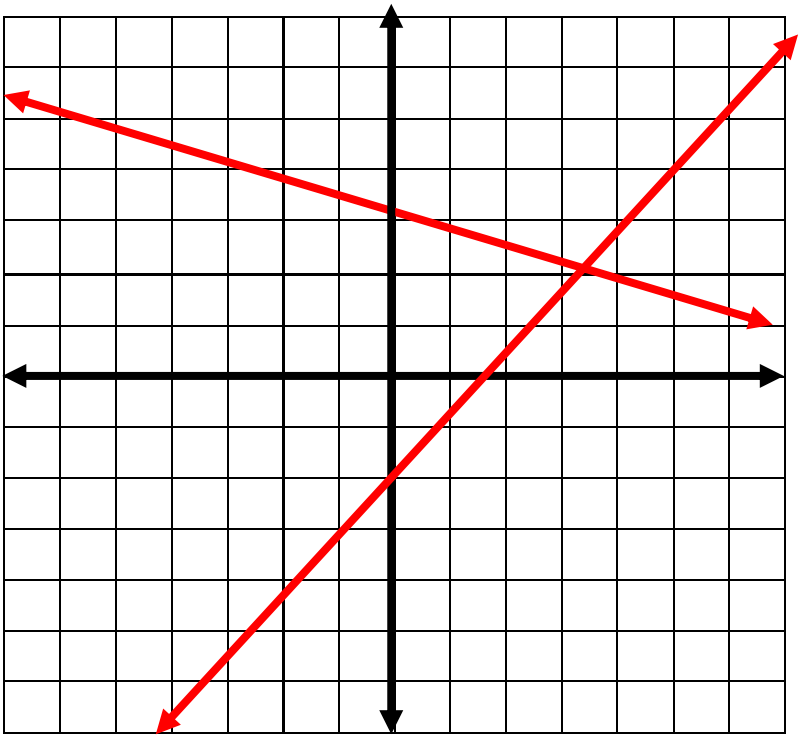
The slopes ($\frac{rise}{run}$) of these two lines are the same.

Two perpendicular lines form a right angle.



The slopes ($\frac{rise}{run}$) of these two lines are exact opposite reciprocals of each other.

These two lines are neither parallel nor perpendicular.



The slopes ($\frac{rise}{run}$) of these two lines are neither the same nor reciprocals of each other.

Let's learn a few ways to identify these two different families of linear equations.

1. Determine if two lines are parallel, perpendicular or neither.

A. 1. $y = 4x + 7$ 2. $y = 4x - 18$

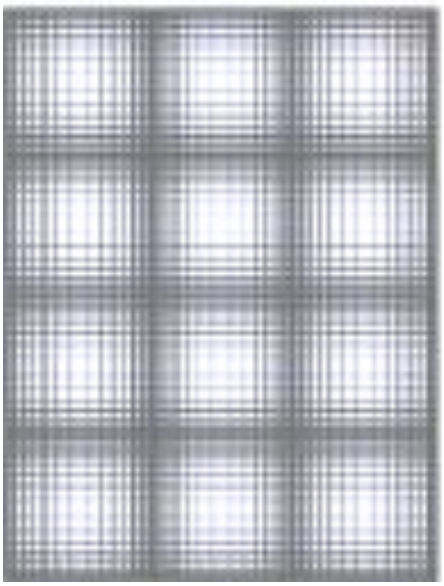
i. Find the slope of the first equation. $m = 4$

ii. Find the slope of the second equation. $m = 4$

iii. Compare the two slopes. They are the same.

These two lines are Parallel because they have the same slopes.

B. 1. $12 = 2x - 3y$ 2. $4 = -3x - 2y$



i. Find the slope of the first equation. $m = \frac{2}{3}$

- 12 = 2x – 3y
- 3y + 12 = 2x
- 3y = 2x – 12
- Y = $\frac{2}{3}$x – 4

ii. Find the slope of the second equation. $m = -\frac{3}{2}$

- 4 = -3x – 2y
- 3x + 4 = - 2y
- $-\frac{3}{2}$x – 2 = y

iii. Compare the two slopes. They are opposite.

These two lines are Perpendicular because they have the reciprocal slopes.

C. 1. $3x - y = 9$ 2. $x + 3y = 36$

i. Find the slope of the first equation. $m = 3$

- 3x – y = 9
- -y = -3x + 9
- Y = 3x – 9

ii. Find the slope of the second equation. $m = -\frac{1}{3}$

- x + 3y = 36
- 3y = -x + 36
- y = $-\frac{1}{3}$x + 12

iii. Compare the two slopes. They are opposite.

These two lines are Perpendicular because they have the reciprocal slopes.

D. 1. $\frac{1}{2}x + y = 8$ 2. $2y = x + 14$

i. Find the slope of the first equation. $m = -\frac{1}{2}$

- $\frac{1}{2}x + y = 8$
- $y = -\frac{1}{2}x + 8$

ii. Find the slope of the second equation. $m = \frac{1}{2}$

- 2y = x + 14
- Y = $\frac{1}{2}$x + 7

iii. Compare the two slopes. They are neither.

These two lines are neither Parallel nor Perpendicular because their slopes are just opposite.

E. 1. $y = -10 + 5x$ 2. $y = 5x + 2$

i. Find the slope of the first equation. $m = 5$

- Y = -10 + 5x
- Y = 5x - 10

ii. Find the slope of the second equation. $m = 5$

- Y = 5x + 2

iii. Compare the two slopes. They are the same.

These two lines are Parallel because they have the same slopes.

2. Determine the equation of the line given one point and a parallel line.

A. Write the slope-intercept form of the equation of the line passing through the point (1, -1) and parallel to the line y = -5x – 4

i. Determine the slope of the given line. m = -5

Remember:

Slope-intercept Form: y = mx + b.

ii. Determine the slope of the parallel line. m = -5

ii. Plug in the given information into the slope-intercept form.

x = 1 and y = -1 -1 = (-5)(1) + b

iii. Solve for the y-intercept of the desired equation.

-1 = -5 + b

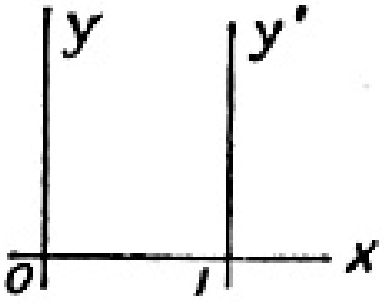
4 = b or b = 4

iv. Write the equation in slope-intercept form. Y = -5x + 4

B. Write the slope-intercept form of the equation of the line passing through the point (2, 0) and parallel to the line y = 2x – 2

i. Determine the slope of the given line. m = 2

ii. Determine the slope of the parallel line. m = 2



ii. Plug in the given information into the slope-intercept form.

x = 2 and y = 0 0 = (2)(2) + b

iii. Solve for the y-intercept of the desired equation.

0 = 4 + b

-4 = b or b = -4

iv. Write the equation in slope-intercept form. Y = 2x - 4

C. Write the slope-intercept form of the equation of the line passing through the point (5, -1) and parallel to the line y = 4x – 1

i. Determine the slope of the given line. m = 4

ii. Determine the slope of the parallel line. m = 4

ii. Plug in the given information into the slope-intercept form.
x = 5 and y = -1 -1 = (4)(5) + b

iii. Solve for the y-intercept of the desired equation.
-1 = 20 + b
-21 = b or b = -21

iv. Write the equation in slope-intercept form. Y = 4x - 21

3. Determine the equation of the line given one point and a perpendicular line.

A. Give the slope-intercept form of the equation of the line that is perpendicular to 8x + 5y = -7 and contains (5, 3).

i. Determine the slope of the given line. m = $-\frac{8}{5}$

- 8x + 5y = -7
- 5y = -8x – 7
- Y = $-\frac{8}{5}$x - $\frac{7}{5}$

ii. Determine the slope of the perpendicular line. m = $\frac{5}{8}$

ii. Plug in the given information into the slope-intercept form.
x = 5 and y = 3 3 = ($\frac{5}{8}$)(5) + b

iii. Solve for the y-intercept of the desired equation.
3 = ($\frac{5}{8}$)(5) + b
3 = $\frac{25}{8}$ + b
$3 - \frac{25}{8} = b$
$-\frac{1}{8} = b \text{ or } b = -\frac{1}{8}$

iv. Write the equation in slope-intercept form. Y = $\frac{5}{8}x - \frac{1}{8}$

B. Give the slope-intercept form of the equation of the line that is perpendicular to 3x + 8y = -8 and contains (9, 7).

i. Determine the slope of the given line. m = $-\frac{3}{8}$

- 3x + 8y = -8
- 8y = -3x – 8
- Y = $-\frac{3}{8}$x - 1

ii. Determine the slope of the perpendicular line. m = $\frac{8}{3}$

ii. Plug in the given information into the slope-intercept form.

x = 9 and y = 7 $7 = (\frac{8}{3})(9) + b$

iii. Solve for the y-intercept of the desired equation.

$7 = (\frac{8}{3})(9) + b$

$3 = 24 + b$

$3 - 24 = b$

$-21 = b \text{ or } b = -21$

iv. Write the equation in slope-intercept form. $Y = \frac{8}{3}x - 21$

C. Give the slope-intercept form of the equation of the line that is perpendicular to 5x + 6y = 2 and contains (-9,- 3).

i. Determine the slope of the given line. m = -1

- 5x + 6y = 2
- 5y = -5x + 2
- Y = -1x + $\frac{2}{5}$

ii. Determine the slope of the perpendicular line. m = 1

ii. Plug in the given information into the Slope-intercept Form.

x = -9 and y = -3 -3 = (1)(-9) + b

iii. Solve for the y-intercept of the desired equation.

-3 = (1)(-9) + b -> -3 = -9 + b

$-3 + 9 = b$ -> $6 = b \text{ or } b = 6$

iv. Write the equation in slope-intercept form. $Y = x + 6$

Parallel & Perpendicular
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Student Practice Worksheet

Name_____Date_____Grade_____

Find the slope-intercept form of the line described.

1. Passes through (4, 2), parallel to y = $-\frac{3}{4}x - 5$

2. Passes through (4, 2), perpendicular to y = $-\frac{3}{4}x - 5$

3. Passes through (-3, -3), parallel to y = $\frac{7}{3}x + 3$

4. Passes through (-3, -3), Perpendicular to y = $\frac{7}{3}x + 3$

5. Passes through (-4, 0), parallel to y =$\frac{3}{4}x - 2$

6. Passes through (-4, 0), perpendicular to y =$\frac{3}{4}x - 2$

7. Passes through (-1, 4), parallel to y = -5x + 2

8. Passes through (-1, 4), perpendicular to y = -5x + 2

9. Passes through (2, 0), perpendicular to y = $\frac{1}{3}x + 3$

10. Passes through (2, 0), parallel to y = $\frac{1}{3}x + 3$

11. Passes through (4, -4), parallel to y = -x – 4

12. Passes through (4, -4), perpendicular to y = -x – 4

13. Passes through (-2, 4), perpendicular to y = $-\frac{5}{2}x + 5$

14. Passes through (-2, 4), parallel to y = $-\frac{5}{2}x + 5$

15. Passes through (-4, -1), parallel to y = $-\frac{1}{2}x - 1$

16. Passes through (-4, -1), perpendicular to y = $-\frac{1}{2}x - 1$

Parallel & Perpendicular Lines

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Student Practice Worksheet

Answer Key

Name_____ Date_____ Grade_____

Find the slope-intercept form of the line described.

1. Passes through (4, 2), parallel to y = $-\frac{3}{4}x - 5$

$y = -\frac{3}{4}x + 5$

2. Passes through (4, 2), perpendicular to y = $-\frac{3}{4}x - 5$

$y = \frac{4}{3}x - \frac{10}{3}$

3. Passes through (-3, -3), parallel to y = $\frac{7}{3}x + 3$

$y = \frac{7}{3}x + 4$

4. Passes through (-3, -3), Perpendicular to y = $\frac{7}{3}x + 3$

$y = -\frac{3}{7}x - \frac{30}{7}$

5. Passes through (-4, 0), parallel to y =$\frac{3}{4}x - 2$

$y = \frac{3}{4}x + 3$

6. Passes through (-4, 0), perpendicular to y =$\frac{3}{4}x - 2$

$y = -\frac{4}{3}x - \frac{16}{3}$

7. Passes through (-1, 4), parallel to y = -5x + 2

$y = -5x - 1$

8. Passes through (-1, 4), perpendicular to y = -5x + 2

$y = \frac{1}{5}x + \frac{17}{4}$

9. Passes through (2, 0), perpendicular to $y = \frac{1}{3}x + 3$

$y = -3x + 6$

10. Passes through (2, 0), parallel to $y = \frac{1}{3}x + 3$

$y = \frac{1}{3}x - \frac{2}{3}$

11. Passes through (4, -4), parallel to y = -x – 4

$y = -x$

12. Passes through (4, -4), perpendicular to y = -x – 4

$y = -x - 8$

13. Passes through (-2, 4), perpendicular to $y = -\frac{5}{2}x + 5$

$y = \frac{2}{5}x + \frac{24}{5}$

14. Passes through (-2, 4), parallel to $y = -\frac{5}{2}x + 5$

$y = -\frac{5}{2}x - 1$

15. Passes through (-4, -1), parallel to $y = -\frac{1}{2}x - 1$

$y = -\frac{1}{2}x - 3$

16. Passes through (-4, -1), perpendicular to $y = -\frac{1}{2}x - 1$

$y = 2x + 7$

Parallel & Perpendicular Lines
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Student Practice Worksheet
Rubric

Criteria					
	4	3	2	1	0
Mechanics	No math errors	No major math errors or serious flaws in reasoning.	May be some serious math error or flaws in reasoning.	Major math errors or serious flaws in reasoning.	Blank answers

Quiz Grading Rubric:

Problem	Total points of Correct Answer		Problem	Total points of Correct Answer	
1.			11.		
2.			12.		
3.			13.		
4.			14.		
5.			15.		
6.			16.		
7.					
8.					
9.					
10.					

Parallel & Perpendicular Lines

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Quiz

Name_____Date_____Grade_____

Find the slope-intercept form of the line described.

1. Passes through (-3, -3), parallel to y = $\frac{7}{3}x + 3$

2. Passes through (-2, 4), perpendicular to y = $-\frac{5}{2}x + 5$

3. Passes through (-4, 0), perpendicular to y =$\frac{3}{4}x - 2$

4. Passes through (2, 0), parallel to y = $\frac{1}{3}x + 3$

5. Passes through (-4, -1), perpendicular to y = $-\frac{1}{2}x - 1$

6. Passes through (4, 2), parallel to y = $-\frac{3}{4}x - 5$

Parallel & Perpendicular Lines

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Quiz

Answer Key

Name_____Date_____Grade_____

Find the slope-intercept form of the line described.

1. Passes through (-3, -3), parallel to $y = \frac{7}{3}x + 3$

$y = -\frac{3}{7}x - \frac{30}{7}$

2. Passes through (-2, 4), perpendicular to $y = -\frac{5}{2}x + 5$

$y = -\frac{5}{2}x - 1$

3. Passes through (-4, 0), perpendicular to $y = \frac{3}{4}x - 2$

$y = -\frac{4}{3}x - \frac{16}{3}$

4. Passes through (2, 0), parallel to $y = \frac{1}{3}x + 3$

$y = \frac{1}{3}x - \frac{2}{3}$

5. Passes through (-4, -1), perpendicular to $y = -\frac{1}{2}x - 1$

$y = -\frac{1}{2}x - 3$

6. Passes through (4, 2), parallel to $y = -\frac{3}{4}x - 5$

$y = -\frac{3}{4}x + 5$

Parallel & Perpendicular Lines
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Quiz
Rubric

Criteria					
	4	3	2	1	0
Mechanics	No math errors	No major math errors or serious flaws in reasoning.	May be some serious math error or flaws in reasoning.	Major math errors or serious flaws in reasoning.	Blank answers

Quiz Grading Rubric:

Problem	Total points of Correct Answer		Problem	Total points of Correct Answer	
1.			6.		
2.					
3.					
4.					
5.					