Lines, Lines, Lines!!!

Standard Form of a Linear Equation

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Lesson Plan

I. Topic: Standard Form

II. Goals and Objectives:
   A. The students will convert equations into standard form.
   B. The students will write equations in standard form.
   C. The students will find the coefficients of the equations.

III. Massachusetts Learning Standards:

1. 8.P.6
   Identify the roles of variables within an equation, e.g., \( y = mx + b \), expressing \( y \) as a function of \( x \) with parameters \( m \) and \( b \).

2. 8.P.7
   Set up and solve linear equations and inequalities with one or two variables, using algebraic methods, models, and/or graph.

3. 10.P.2
   Demonstrate an understanding of the relationship between various representations of a line. Determine a line’s slope and \( x \)- and \( y \)-intercepts from its graph or from a linear equation that represents the line. Find a linear equation describing a line from a graph or geometric description of the line, e.g, by using the “point-slope” or “slope \( y \)-intercept” formulas. Explain the significance of a positive, negative, zero, or undefined slope.

4. 10.P.8
   Solve everyday problems that can be modeled using systems of linear equations or inequalities. Apply algebraic and graphical methods to the solution. Use technology when appropriate. Include mixture, rate, and work problems.

IV. Materials:
   A. Whiteboard with dry-erase markers (Blackboard with chalk could also be used.).
   B. Ruler.
   C. Pencils.
   D. Calculator.
   E. Graphing paper.
   F. Standard Form Worksheets (Practice Worksheet, Quiz Worksheet).
V. Presentation Outline:
   A. Introductions. "The Standard Form of a line."
      I. Equation
      II. Definition
   B. Write an equation in Standard Form from the given linear equation.
      Example
   C. Write an equation in Standard Form given a point and a Slope
      Example
   D. Write an equation in Standard Form given two points.
      Example

VI. Presentation:
   A. Presentation Notes
   B. Power Point Presentation

VII. Independent Practice: Standard Form Worksheet
   A. Class work: # 1 - 29 Odds
   B. Homework: #s 2 - 30 Evens
   C. Due 2 days from given day. Allow students to complete those questions which they did not complete in class.

VIII. Topic Assessment: Standard Form Quiz
   A. Answer questions from homework.
   B. 5 Question Quiz: 10 – 15 minutes
   C. 10 Question Quiz: 15 – 20 minutes
Lines, Lines, Lines!!!

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Standard Form of a Linear Equation

Introduction
All of the linear equations we have studied so far can be written in Standard Form. The Standard form of a linear equation helps us identify its coefficients along with an easiness of graphing a line by finding the x- and y- intercept of such a line. The Standard Form is acquired when all of the criteria are met.

Definitions and Formula
The Standard Form of a linear equation is $Ax + By = C$. However, each of the following criteria must be met:

- $A$, $B$, and $C$ are integers.

Remember:
Integers are positive and negative numbers including zero which do not have fractions or decimals. i.e. $(-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5...)$.

- $A$, $B$, and $C$'s greatest common factor is 1.

Remember:
When two numbers have a greater common factor of 1, then they cannot be divided by any other number than one. i.e. $(3$ and $5), (10$ and $7), (3, 9,$ and $8), (-4, 11)$.

- 'A' is greater than or equal to Zero ($A \geq 0$).

- $A$ and $B$ are not both Zero.
**Key Concepts**

It is important to learn how to recognize when an equation is in the Standard form and when it is not.

The following lines are in Standard Form

A. \(5x - 7y = 6\) \(\Leftrightarrow\) Where \(A = 5\), \(B = -7\), and \(C = 6\)

B. \(x + 3y = 0\) \(\Leftrightarrow\) Where \(A = 1\), \(B = 3\), and \(C = 0\)

C. \(1000x - 10000y = 1\) \(\Leftrightarrow\) Where \(A = 1000\), \(B = 10,000\), and \(C = 1\)

D. \(y = -3\) \(\Leftrightarrow\) Where \(A = 0\), \(B = 1\), and \(C = 4\)

The following lines are not in Standard Form:

A. \(3y = -7x + 12\) \(\Leftrightarrow\) 'A' and 'B' must be on the same side.

B. \(-3x + y = 5\) \(\Leftrightarrow\) 'A' must be greater than or equal to Zero.

C. \(7x + 14y = 21\) \(\Leftrightarrow\) These numbers can be divided by 7.

D. \(\frac{2}{3}x - y = -3\) \(\Leftrightarrow\) \(\frac{2}{3}\) is not an integer.

E. \(0x + 0y = 4\) \(\Leftrightarrow\) 'A' and 'B' can't both be Zero.

The following lines are in the form of other equations which we will learn or have learned:

A. \(y = mx + b\) \(\Leftrightarrow\) Slope-Intercept Form

B. \(y - y_1 = m(x - x_1)\) \(\Leftrightarrow\) Point-Slope Form

C. \(Ax + By = C\) \(\Leftrightarrow\) Standard Form

D. \(y = b\) \(\Leftrightarrow\) Horizontal Line (Zero Slope)

E. \(x = b\) \(\Leftrightarrow\) Vertical Line (Undefined Slope)
Let’s see a few ways to use or find the Standard Form of a linear equation.

1. Write an equation in Standard Form from the given linear equation.

   A. \( y = -4x + 8 \)
      
      i. Original equation. \( y = -4x + 8 \)
      
      ii. Add \(-4x\) to both sides. \(4x + y = -4x + 4x + 8\)
      
      iii. Simplify. \(4x + y = 8\)
      
      \(A = 4, B = 1, \text{ and } C = 8\)

   B. \(8 + 3x = -9y\)
      
      i. Original equation. \(8 + 3x = -9y\)
      
      ii. Subtract \(8\) from both sides. \(8 - 8 + 3x = -9y - 8\)
      
      iii. Simplify. \(3x = -9y - 8\)
      
      iv. Add \(-9y\) to both sides. \(3x + 9y = -9y + 9y - 8\)
      
      v. Simplify. \(3x + 9y = -8\)
      
      \(A = 3, B = 9, \text{ and } C = -8\)

   C. \(y = \frac{3}{5}x + 5\)
      
      i. Original equation \(y = \frac{3}{5}x + 5\)
      
      ii. Multiply all terms by \(5\). \(5 \cdot (y = \frac{3}{5}x + 5)\)
      
      iii. Simplify. \(5y = 3x + 25\)
      
      iv. Subtract \(3x\) from both sides. \(-3x + 5y = 3x - 3x + 25\)
v. Simplify. \(-3x + 5y = 25\)

vi. Multiply all terms by -1. \(-1 \cdot (-3x + 5y = 25)\)

vii. Simplify. \(3x - 5y = -25\)

\[ A = 3, \ B = -5, \text{ and } C = -25 \]

D. \(\frac{2}{3}x - \frac{1}{4}y = 6\)

i. Original equation. \(\frac{2}{3}x - \frac{1}{4}y = 6\)

ii. Multiply all terms by 12. \(12 \cdot \left(\frac{2}{3}x - \frac{1}{4}y = 6\right)\)

iii. Simplify. \(8x - 3y = 72\)

\[ A = 8, \ B = -3, \text{ and } C = 72 \]

2. Write an equation in Standard Form given a point and a slope.

Note: It is always best to used the Point-Slope Form of an equation when the only information given is one point and the slope of the linear equation.

A. \(m = \frac{1}{2}\) and point (3, 4)

i. Point-Slope Form. \(y - y_1 = m(x - x_1)\)

ii. Substitute the information given. \(y - 4 = \frac{1}{2}(x - 3)\)

iii. Simplify using the Distributive Property. \(y - 4 = \frac{1}{2}x - \frac{3}{2}\)

iv. Multiply all terms by 2. \(2 \cdot (y - 4 = \frac{1}{2}x - \frac{3}{2})\)

v. Simplify. \(2y - 8 = x - 3\)
vi. Subtract both by sides by $x$. \[-x + 2y - 8 = x - x - 3\]

vii. Simplify. \[-x + 2y - 8 = -3\]

viii. Add 8 to both sides. \[-x + 2y - 8 + 8 = -3 + 8\]

ix. Simplify. \[-x + 2y = 5\]

x. Multiply all terms by -1. \[-1(-x + 2y = 5)\]

xi. Simplify. \[x - 2y = -5\]

\[A = 1, B = -2, \text{ and } C = -5\]

B. \[m = 1 \text{ and point } (-2, 1)\]

i. Point-Slope Form \[y - y_1 = m(x - x_1)\]

ii. Substitute the information given. \[y - 1 = 1(x + 2)\]

iii. Simplify using the Distributive Property. \[y - 1 = x + 2\]

iv. Subtract both by sides by $x$. \[-x + y - 1 = x - x + 2\]

v. Simplify. \[-x + y - 1 = 2\]

vi. Add 1 to both sides. \[-x + y - 1 + 1 = 2 + 1\]

vii. Simplify. \[-x + y = 3\]

viii. Multiply all terms by -1. \[-1(-x + y = 3)\]

ix. Simplify. \[x - y = -3\]

\[A = 1, B = -1, \text{ and } C = -3\]
C. \( m = -3 \) and point \((0, -8)\)

i. Point-Slope Form \[ y - y_1 = m(x - x_1) \]

ii. Substitute the information given. \[ y + 8 = -3(x - 0) \]

iii. Simplify using the Distributive Property. \[ y + 8 = -3x + 0 \]

iv. Simplify. \[ y + 8 = -3x \]

v. Subtract both by sides by \(8\). \[ y + 8 - 8 = -3x - 8 \]

vi. Simplify. \[ y = -3x - 8 \]

vii. Add \(3x\) to both sides. \[ 3x + y = -3x + 3x - 8 \]

viii. Simplify. \[ 3x + y = -8 \]

\[ A = 3, \ B = 1, \ and \ C = -8 \]

3. Write an equation in Standard Form given two points.

**Note:**

(The following examples were used on the previous lesson on Slope-Intercept Form)

A. Write an equation of the line in Standard Form which passes through \((-1, 4)\) and \((1, -2)\).

i. Find the slope between the two points. \[ m = \frac{-2-4}{1-(-1)} = \frac{-6}{2} = -3 \]

ii. Find the value of the y-intercept (\(b\)) by substituting the value of one of the points into the equation. \[ 4 = -3(-1) + b \]

iii. Simplify and solve. \[ 4 = 3 + b \ so \ b = 1 \]

iv. Substitute the value of 'b' into the equation. \[ y = -3x + 1 \]

v. Add \(-3x\) to both sides. \[ 3x + y = -3x + 3x + 1 \]

vi. Simplify. \[ 3x + y = 1 \]

\[ A = 3, \ B = 1 \ and \ C = 1 \]
B. Write an equation of the line in Standard Form which passes through (5, -3) and (-4, 3).

i. Find the slope between the two points.  
   \[ m = \frac{3 - (-3)}{-4 - 5} = \frac{6}{-9} = -\frac{2}{3} \]

ii. Find the value of the y-intercept (b) by substituting the value of one of the points into the equation.  
   \[ -3 = -\frac{2}{3} (5) + b \]

iii. Simplify and solve.  
   \[ -3 = -\frac{10}{3} + b \text{ so } b = \frac{1}{3} \]

iv. Substitute the value of 'b' into the equation.  
   \[ y = -\frac{2}{3}x + \frac{1}{3} \]

v. Add \( \frac{2}{3}x \) to both sides.  
   \[ \frac{2}{3}x + y = -\frac{2}{3}x + \frac{2}{3}x + \frac{1}{3} \]

vi. Simplify.  
   \[ \frac{2}{3}x + y = \frac{1}{3} \]

vii. Multiply all terms by 3.  
   \[ 3 \cdot \frac{2}{3}x + 3 \cdot y = 3 \cdot \frac{1}{3} \]

viii. Simplify.  
   \[ 3x + 3y = 1 \]

   \[ A = 3, B = 3 \text{ and } C = 1 \]

C. Write an equation of the line in Standard Form which passes through (-1, 2) and (3, 4).

i. Find the slope between the two points.  
   \[ m = \frac{4 - 2}{3 - (-1)} = \frac{2}{4} = \frac{1}{2} \]

ii. Find the value of the y-intercept (b) by substituting the value of one of the points into the equation.  
   \[ 2 = \frac{1}{2}(-1) + b \]

iii. Simplify and solve.  
   \[ 2 = -\frac{1}{2} + b \text{ so } b = \frac{5}{2} \]
Lines, Lines, Lines!!! Standard Form of a Linear Equation

iv. Substitute the value of 'b' into the equation.
\[ y = \frac{1}{2}x + \frac{5}{2} \]

v. Subtract \( \frac{1}{2}x \) from both sides.
\[ -\frac{1}{2}x + y = \frac{1}{2}x - \frac{1}{2}x + \frac{5}{2} \]

vi. Simplify.
\[ -\frac{1}{2}x + y = \frac{5}{2} \]

vii. Multiply all terms by -2.
\[ -2 \cdot -\frac{1}{2}x + (-2) \cdot y = -2 \cdot \frac{5}{2} \]

viii. Simplify.
\[ 1x - 2y = -5 \]

A = 1, B = -2 and C = -5
Standard Form of a Line

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Student Practice Worksheet

Name____________________________________________Date______________Grade___________

Write an equation in Standard Form given a line in slope-intercept form.

1. \( y = \frac{7}{10}x + \frac{4}{5} \)

2. \( y = -\frac{4}{5}x - 1 \)

3. \( y = -\frac{2}{5}x + \frac{3}{10} \)

4. \( y = x - \frac{4}{7} \)

5. \( y = -\frac{3}{7}x - \frac{4}{7} \)

6. \( y = \frac{1}{5}x + 1 \)

7. \( y = -x + \frac{1}{6} \)

8. \( y = -\frac{1}{5}x - \frac{7}{10} \)

9. \( y = \frac{2}{5}x - \frac{2}{5} \)

10. \( y = \frac{4}{7}x + \frac{5}{7} \)

Write an equation in Standard Form given two points.

11. (-3, 1) and (5, 4)

12. (5, -6) and (2, 3)
13. (2, -2) and (-6, 1)  
14. (3, 4) and (-7, 4)

15. (0, -2) and (7, 0)  
16. (-5, -1) and (4, -7)

17. (1, 0) and (4, 1)  
18. (-2, 0) and (0, -2)

19. (-1, -2) and (1, 4)  
20. (-2, 5) and (-1, 2)

Write an equation in Standard Form given a point and a Slope.

21. (2, 3), m = 3  
22. (-4, 7), m = 3

23. (0, 3), m = -4  
24. (7, 2), m = -5

25. (3, 2), m = \frac{2}{3}  
26. (2, -3), m = -\frac{3}{2}

27. (1, 2), m = 7  
28. (3, -1), m = -1

29. (-2, 5), m = -4  
30. (3, 5), m = \frac{5}{3}
Write an equation in Standard Form given a line in slope-intercept form.

1. \( y = \frac{7}{10}x + \frac{4}{5} \)

\( 7x - 10y = -8 \)

2. \( y = -\frac{4}{5}x - 1 \)

\( 4x + 5y = -5 \)

3. \( y = -\frac{2}{5}x + \frac{3}{10} \)

\( 4x + 10y = 3 \)

4. \( y = x - \frac{4}{7} \)

\( 7x + 7y = 4 \)

5. \( y = -\frac{3}{7}x - \frac{4}{7} \)

\( 3x + 7y = -4 \)

6. \( y = \frac{1}{5}x + 1 \)

\( x - 5y = -5 \)

7. \( y = -x + \frac{1}{6} \)

\( 6x + 6y = 1 \)

8. \( y = -\frac{1}{5}x - \frac{7}{10} \)

\( 2x + 10y = -7 \)

9. \( y = \frac{2}{5}x - \frac{2}{5} \)

\( 2x - 5y = 2 \)

10. \( y = \frac{4}{7}x + \frac{5}{7} \)

\( 4x - 7y = -5 \)

Write an equation in Standard Form given two points.

11. \((-3, 1)\) and \((5, 4)\)

\( 3x - 8y = -17 \)

12. \((5, -6)\) and \((2, 3)\)

\( 3x + y = 9 \)
13. (2, -2) and (-6, 1)
   \[3x + 8y = -10\]

14. (3, 4) and (-7, 4)
   \[y = 4\]

15. (0, -2) and (7, 0)
   \[2x - 7y = 14\]

16. (-5, -1) and (4, -7)
   \[2x + 3y = -7\]

17. (1, 0) and (4, 1)
   \[x - 3y = 1\]

18. (-2, 0) and (0, -2)
   \[x + y = -2\]

19. (-1, -2) and (1, 4)
   \[3x - y = -1\]

20. (-2, 5) and (-1, 2)
   \[3x + y = -1\]

**Write an equation in Standard Form given a point and a Slope.**

21. (2, 3), \(m = 3\)
   \[3x - y = 3\]

22. (-4, 7), \(m = 3\)
   \[3x - y = -19\]

23. (0, 3), \(m = -4\)
   \[4x + y = 3\]

24. (7, 2), \(m = -5\)
   \[5x + y = 37\]

25. (3, 2), \(m = \frac{2}{3}\)
   \[2x - 3y = 0\]

26. (2, -3), \(m = \frac{3}{2}\)
   \[3x + 2y = 0\]

27. (1, 2), \(m = 7\)
   \[7x - y = 5\]

28. (3, -1), \(m = -1\)
   \[x + y = 2\]

29. (-2, 5), \(m = -4\)
   \[4x + y = -3\]

30. (3, 5), \(m = \frac{5}{3}\)
   \[5x - 3y = 0\]
Standard Form of a Line

Student Practice Worksheet
Rubric

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Quiz Grading Rubric:

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Total: _______________
Write an equation in Standard Form given a line in slope-intercept form.

1. \( y = -2x - 18 \)
2. \( \frac{2}{5}x + 3y = 10 \)
3. \( 10 - 2x = 8y \)
4. \( y = \frac{1}{7}x - 1 \)
5. \( y = \frac{1}{2}(10x + 8) \)

Write an equation in Standard Form given a point and a slope.

6. \( (2, -4), \text{ slope } = -1 \)
7. \( (3, 1), \text{ slope } = \frac{1}{2} \)
8. \( (-1, 2), \text{ slope } = 2 \)
9. \( (-4, 0), \text{ slope } = \frac{3}{4} \)
10. \( (-3, -3), \text{ slope } = \frac{7}{3} \)
Write an equation in Standard Form given two points.

11. (4, -2) and (8, 3)  
12. (2, 10) and (8, 7)  

13. (7, 3) and (8, 5)  
14. (5, 7) and (8, 5)  

15. (10, 3) and (7, 9)
Standard Form of a Line

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Quiz

Answer Key

Name____________________________________________Date________________ Grade___________

Write an equation in Standard Form given a line in slope-intercept form.

1. \( y = -2x - 18 \)

2. \( \frac{2}{5}x + 3y = 10 \)

\( 2x + y = -18 \)

\( 2x + 15y = 50 \)

3. \( 10 - 2x = 8y \)

4. \( y = \frac{1}{7}x - 1 \)

\( x + 4y = 5 \)

\( x - 7y = 7 \)

5. \( y = -\frac{1}{2}(10x + 8) \)

\( 5x + y = -4 \)

Write an equation in Standard Form given a point and a slope.

6. \((2, -4), \text{ slope } = -1\)

7. \((3, 1), \text{ slope } = \frac{1}{2}\)

\( x + y = -2 \)

\( x - 2y = 1 \)

8. \((-1, 2), \text{ slope } = 2\)

9. \((-4, 0), \text{ slope } = \frac{3}{4}\)

\( 2x - y = 4 \)

\( 3x - 4y = -12 \)

10. \((-3, -3), \text{ slope } = \frac{7}{3}\)

\( 7x - 3y = -12 \)
Write an equation in Standard Form given two points.

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