Lines, Lines, Lines!!!
Slopes of a Line
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Lesson Plan

I.   Topic: Slopes

II.   Goals and Objectives:
A. The students will demonstrate understanding of the Slope Formula.
B. The students will learn how to find the slope of a line.
C. The students will learn how to determine line relationship.
D. The students will distinguished the difference between a *positive slope, negative slope, zero slope,* and *undefined slope.*

III.   Massachusetts Learning Standards:

1.   8.P.1
Extend, represent, analyze, and generalize a variety of patterns with tables, graphs, words, and when possible, symbolic representation.

2.   8.P.2
Evaluate simple algebraic expressions for given variable values.

3.   8.P.5
Identify the slope of a line as a measure of its steepness and as a constant rate of change from its table of values, equation, or graph. Apply the concept of slope to the solution of problems.

4.   8.P.6
Identify the roles of variables within an equation, e.g., $y = mx + b$, expressing y as a function of x with parameters m and b.

5.   10.N.2
Simplify numerical expressions, including those involving positive integer, exponents or the absolute value and apply such simplifications in the solutions of problems.

IV.   Materials:
A. Whiteboard with dry-erase markers. (Blackboard with chalk could also be used.)
B. Ruler.
C. Pencils.
D. Calculator.
E. Graphing Paper.
E. Slopes Worksheets (Practice Worksheet, Quiz Worksheet).

V.   Presentation Outline:
A.   Introduction "What are *slopes*? Where are they found?"

B.   Formula.

C.   Key Concepts.

D.   Find the slope between two points.
            I. Formula.
            II. Graph.
            II. Examples.

E.   Determine the slope of a line in the coordinate plane.
            I. Graph.
            II. Examples.

F.   Find a point - given a point and a slope.
            I. Examples.

VI.   Presentation:
A. Presentation Notes.
B. Power Point Presentation.

VII.   Independent Practice: Slopes Worksheet
A. Class work: # 2 - 20 Even

B. Homework: #s 1 - 19 Odds

C. Due 2 days from the day assigned. Allow students to complete those questions which they did not complete in class.

VIII.   Topic Assessment: Slopes Quiz
A. Answer questions from homework.

B. 5 Question Quiz: 10 – 15 minutes

C. 10 Question Quiz: 15 – 20 minutes.

Lines, Lines, Lines!!!

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Slope of a line

Introduction

Slopes are found everywhere; from the inclination of a mountain to the steepness of a road; from the graph of a bank statement to the depth of the ocean floor on a given area.

When a car is going up a mountain, we know the steepness of the mountain by dividing the amount the road rises by the corresponding horizontal distance. The answer is the slope of the mountain.

Definitions

The slope of a line is the ratio of the change in y-coordinates (vertical direction) to the change in x-coordinates (horizontal direction).

In other words, if the bottom of the mountain can be marked with the point (x_1, y_1) and the top of the mountain can be mark with the point (x_2, y_2), where the x's represent the horizontal direction and the y's the vertical direction, then we would know the slope of the mountain by subtracting the ys and the answer dividing it by the subtraction of the xs.

The 'm' (lower case m) is the symbol used to represents the slope between two points. m = Slope.

Slope Formula

The formula for finding the slope between two points in represented in the following way:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

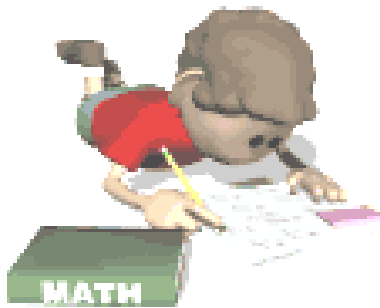This is always true, as long as $x_1 \neq x_2$.
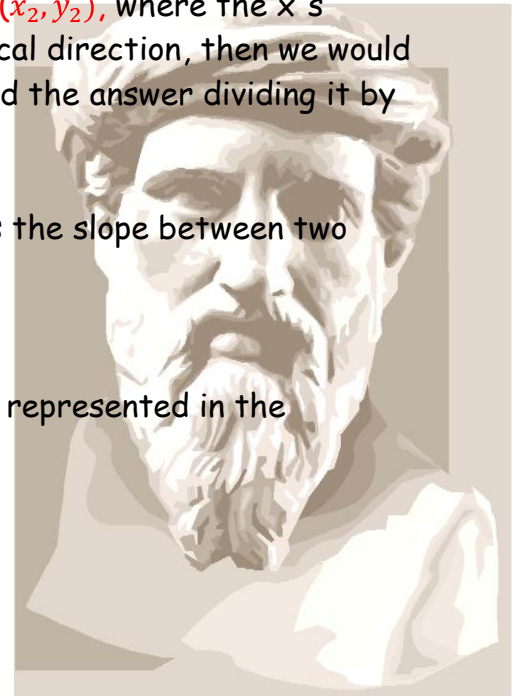
You will hear the slope being refer to in many different ways.

$$m = \frac{Change\ of\ y\ or\ \Delta y}{Change\ of\ x\ or\ \Delta x} = \frac{Rise}{Run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{+\uparrow\ \ -\downarrow}{+\rightarrow\ \ -\leftarrow}$$
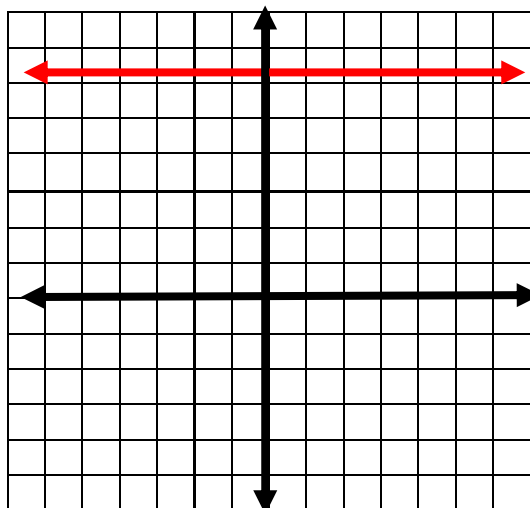
Key Concepts

Other important key concepts about slopes are how they are classified.

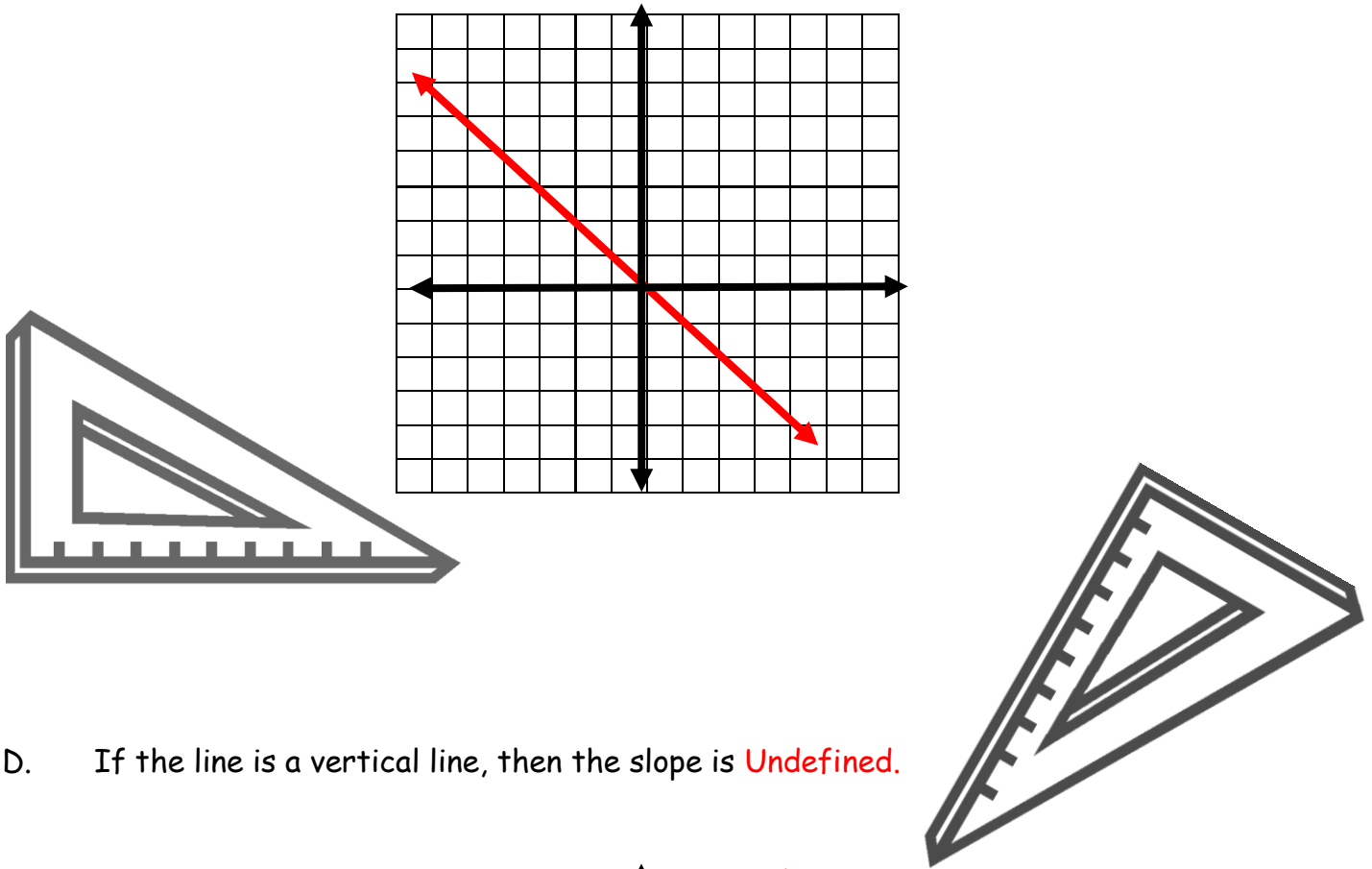A.   If the line going from left to right is rising, then we say the slope has a Positive Slope.
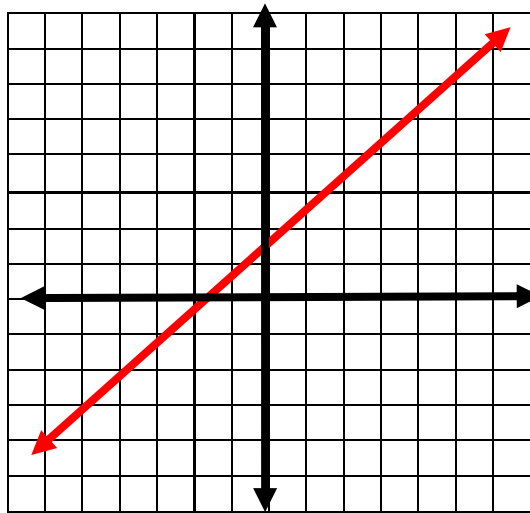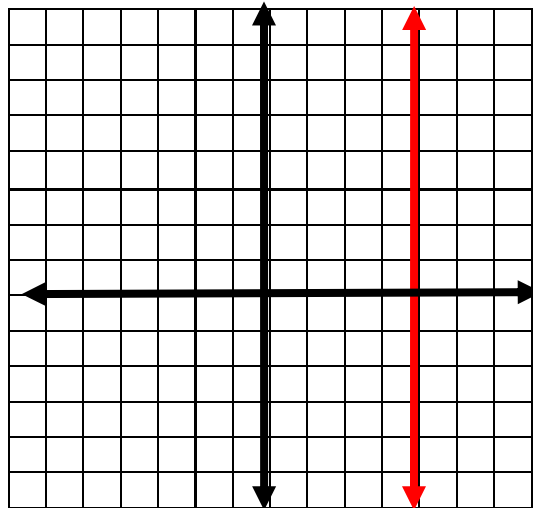
B.   If the Line is horizontal, then the slope is Zero.

C.   If the line going from left to right is falling, then we say the slope is Negative.

D.   If the line is a vertical line, then the slope is Undefined.

Note:

We will learn more about Zero and Undefined Slopes during in the next few lessons.

Let's see a few ways in which to use the Slope Formula.

1. Finding the Slope between two points.

A. Find the slope of the line that passes through (-1, 4) and (1, -2).

I. Let (-1, 4) = (x_1, y_1) and (1, -2) = (x_2, y_2)

II. Remember and re-write the slope formula. $m = \frac{y_2 - y_1}{x_2 - x_1}$

III. Substitute the values of the given question. $m = \frac{-2-4}{1-(-1)}$

IV. Simplify as much as possible. $m = \frac{-6}{2}$

V. Divide whenever possible. $m = -3$

B. Find the slope of the line that passes through (5, -3) and (-4, 3).

I. Let (5, -3) = (x_1, y_1) and (-4, 3) = (x_2, y_2)

II. Remember and re-write the slope formula. $m = \frac{y_2 - y_1}{x_2 - x_1}$

III. Substitute the values of the given question. $m = \frac{3-(-3)}{-4-5}$

IV. Simplify as much as possible. $m = \frac{6}{-9}$

V. Divide or reduce whenever possible. $m = -\frac{2}{3}$

C. Find the slope of the line that passes through (-1, 2) and (3, 4).

I. Let (-1, 2) = (x_1, y_1) and (3, 4) = (x_2, y_2)

II. Remember and re-write the slope formula. $m = \frac{y_2 - y_1}{x_2 - x_1}$

III. Substitute the values of the given question. $m = \frac{4-2}{3-(-1)}$
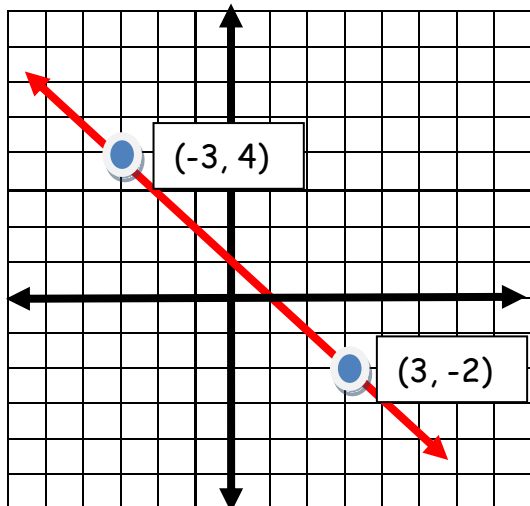
IV. Simplify as much as possible. $m = \frac{2}{4}$

V. Divide or reduce whenever possible. $m = \frac{1}{2}$
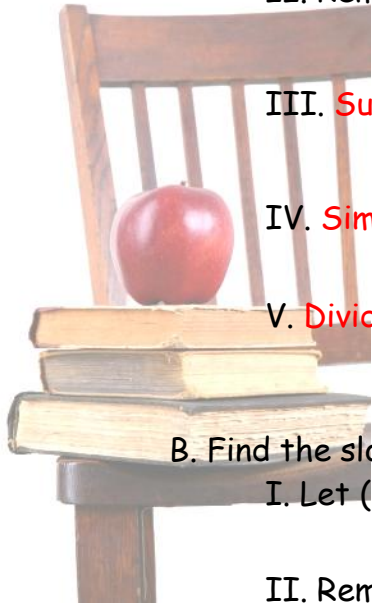
2. Determine the slope of each line.

A.

I. Choose two points on the line.

Let (x_1, y_1)= (3, -2) and $(x_2, y_2) = (-3, 4)$

II. Use the slope formula $\qquad m = \frac{y_2 - y_1}{x_2 - x_1}$

III. Substitute the values. $\qquad m = \frac{4 - (-2)}{-3 - 3}$
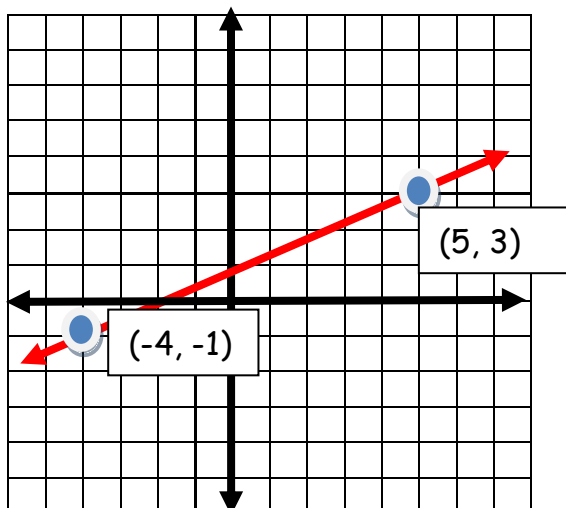
IV. Simplify. $\qquad m = \frac{6}{-6}$

V. Divide or reduce. $\qquad m = -1$

Note:

Any two points on the same line would produce the same response. Also, it doesn't matter which of the points is chosen as (x_1, y_1) or (x_2, y_2); the result will be the same.

B.

I. Choose two points on the line.

Let (x_1, y_1)= (-4, -1) and $(x_2, y_2) = (5, 3)$

II. Use the slope formula $\qquad m = \frac{y_2 - y_1}{x_2 - x_1}$

III. Substitute the values. $\qquad m = \frac{3 - (-1)}{5 - (-4)}$

IV. Simplify. $\qquad m = \frac{4}{9}$

3. Find a point - Given a point and a slope.

A. Find the value of n so that the line through (n, 6) and (10, -3) has a slope of $-\frac{3}{2}$.

I. Let (n, 6) = (x_1, y_1), (10, -3) = (x_2, y_2) and m = $-\frac{3}{2}$.

II. Remember and re-write the slope formula. $m = \frac{y_2 - y_1}{x_2 - x_1}$

III. Substitute the values of the given question. $-\frac{3}{2} = \frac{-3-6}{10-n}$

IV. Subtract or simplify as much as possible. $-\frac{3}{2} = \frac{-9}{10-n}$

V. Use cross-multiplication to solve the fractions. $-3(10 - n) = 2(-9)$

VI. Simplify using the distributive property. $-30 + 3n = -18$

VII. Use addition to isolate the variable. $-30 + 3n + 30 = -18 + 30$

VIII. Simplify as much as possible. $3n = 12$

IX. Divide both sides by 3 to isolate the variable. $\frac{3n}{3} = \frac{12}{3}$

X. Simplify. $n = 4$

Note:

Never lose a moment to re-assure your students that many of these steps can be avoided and that 10 steps are not necessary in all cases.

B. Find the value of r so that the line through (6, -2) and (r, -6) has a slope of 4.

I. Let (6, -2) = (x_1, y_1), (r, -6) = (x_2, y_2) and m = $\frac{4}{1}$.

II. Remember and re-write the slope formula. $m = \frac{y_2 - y_1}{x_2 - x_1}$

III. Substitute the values of the given question. $\frac{4}{1} = \frac{-6-(-2)}{r-6}$

IV. Subtract or simplify as much as possible. $\frac{4}{1} = \frac{-4}{r-6}$

V. Use cross-multiplication to solve the fractions. $-4 = 4(r - 6)$

VI. Simplify using the distributive property. $\quad -4 = 4r - 24$

VII. Use addition to isolate the variable. $\quad -4 + 24 = 4r - 24 + 24$

VIII. Simplify as much as possible. $\quad 20 = 4r$

IX. Divide both sides by 4 to isolate the variable. $\quad \dfrac{20}{4} = \dfrac{4r}{4}$

X. Simplify. $\quad 5 = r \; or \; r = 5$

C. Find the value of t so that the line through (10, t) and (3, 4) has a slope of $-\frac{2}{7}$.

I. Let (10, t) = (x_1, y_1), (3, 4) = (x_2, y_2) and m = $-\frac{2}{7}$.

II. Remember and re-write the slope formula. $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

III. Substitute the values of the given question. $\quad -\dfrac{2}{7} = \dfrac{4 - t}{3 - 10}$

IV. Subtract or simplify as much as possible. $\quad -\dfrac{2}{7} = \dfrac{4 - t}{-7}$

V. Use cross-multiplication to solve the fractions. $\quad -2(-7) = 7(4 - t)$

VI. Simplify using the distributive property. $\quad 14 = 28 - 7t$

VII. Use subtraction to isolate the variable. $\quad 14 - 28 = 28 - 7t - 28$

VIII. Simplify as much as possible. $\quad -14 = -7t$

IX. Divide both sides by -7 to isolate the variable. $\quad \dfrac{-14}{-7} = \dfrac{-7t}{-7}$

X. Simplify. $\quad 2 = t \; or \; t = 2$
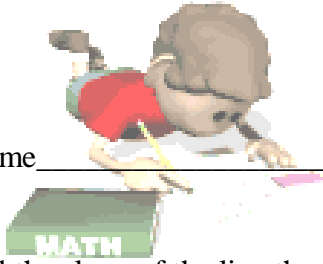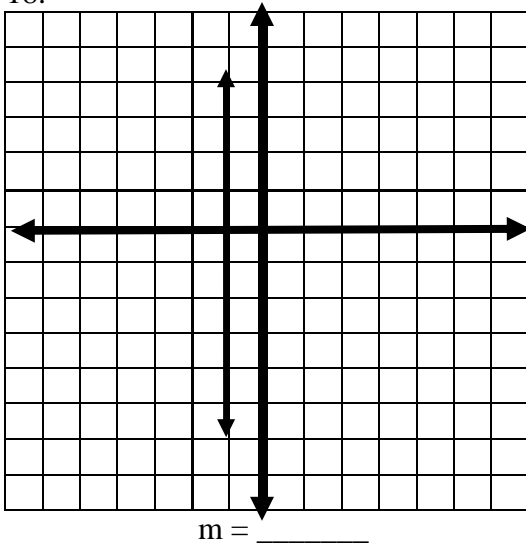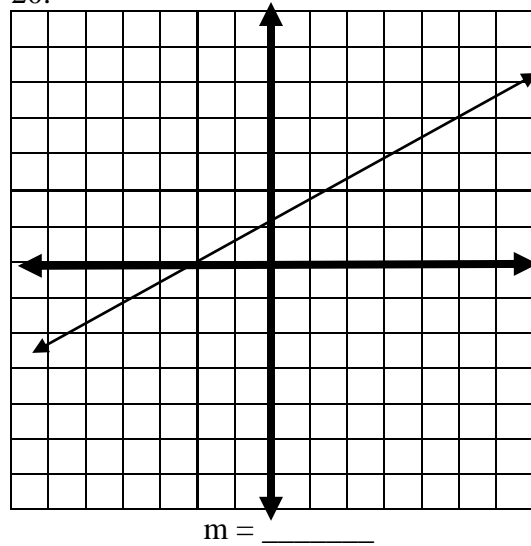
Slopes
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Student Practice Worksheet

Name_____Date_____Grade_____

Find the slope of the line through each pair of given points using the slope formula. $m = \frac{y_2 - y_1}{x_2 - x_1}$

1. (-4, 7), (-6, -4) m = _____   2. (12, 2), (-7, 5) m = _____

3. (3, 0), (-11, -15) m = _____   4. (12, -18), (-15, -18) m = _____

5. (6, 2), (6, -5) m = _____   6. (-3, -7), (-8, -1) m = _____

7. (n, 3), (-1, 0) m = $-\frac{3}{4}$ 8. (3, -5), (s, 0) m = $-\frac{5}{3}$

   n = _____   s = _____

9. (2, r), (4, ¼) m = $-\frac{1}{4}$ 10. (1, 2), (7, t) m = $\frac{7}{6}$

   r = _____   t = _____

Find the slope of the line.

11.

m = _____

14.

m = _____

12.

m = _____

15.

m = _____

13.

m = _____

16.

m = _____

17.

m = _____

19.

m = _____

18.

m = _____

20.

m = _____

Slopes

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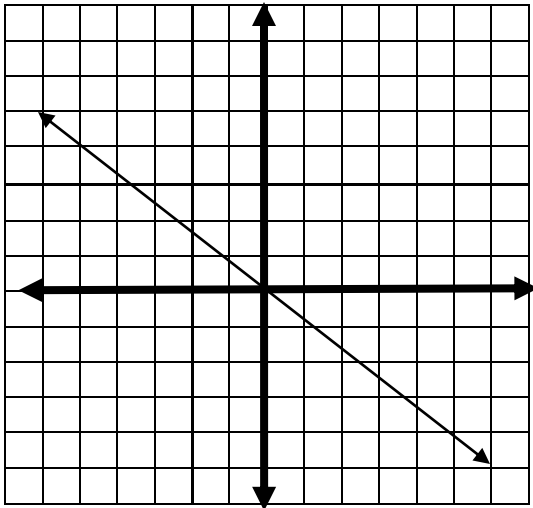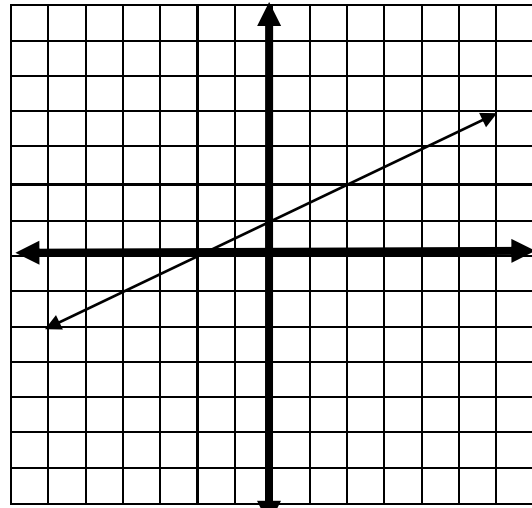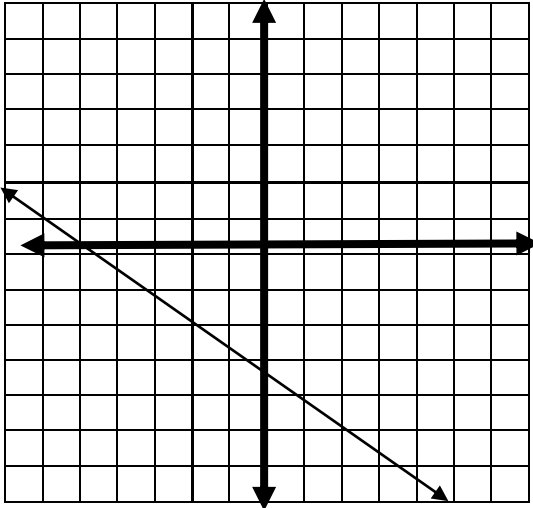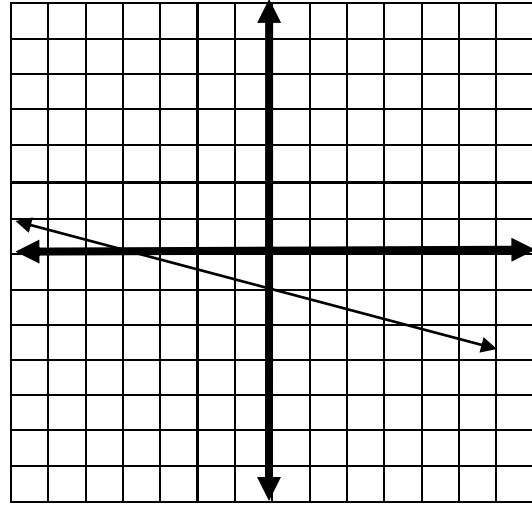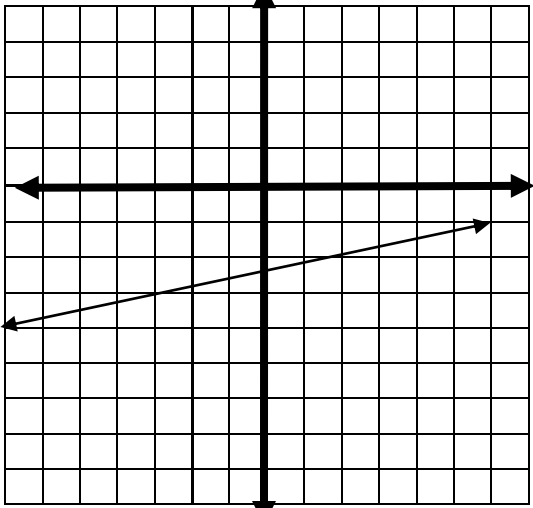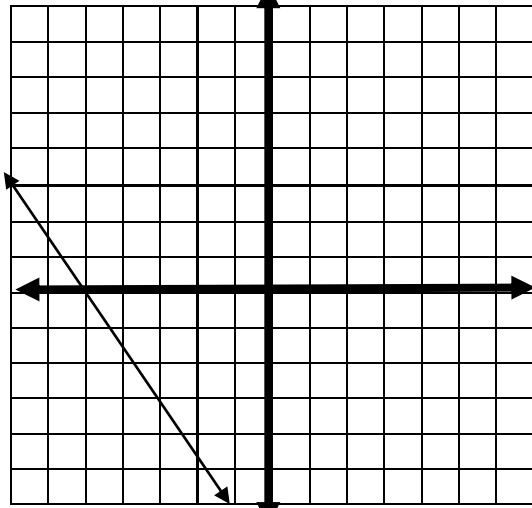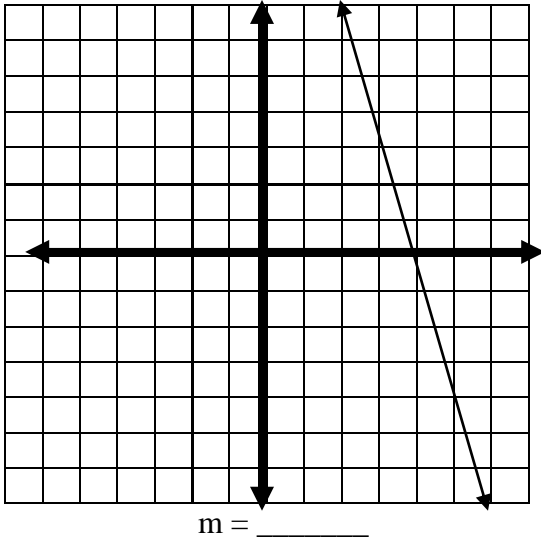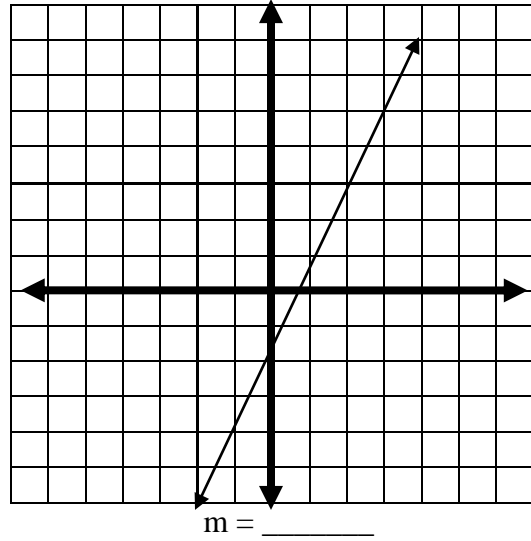
Student Practice Worksheet

Answer Key

Name_____ Date_____ Grade_____

Find the slope of the line through each pair of given points using the slope formula. $m = \frac{y_2 - y_1}{x_2 - x_1}$

1.   (-4, 7), (-6, -4) $m = \frac{11}{2}$

2.   (12, 2), (-7, 5) $m = -\frac{3}{19}$

3.   (3, 0), (-11, -15) $m = \frac{15}{14}$

4.   (12, -18), (-15, -18) $m = 0$

5. (6, 2), (6, -5) $m =$ undefined

6.   (-3, -7), (-8, -1) $m = -\frac{6}{5}$

7.   (n, 3), (-1, 0) $m = -\frac{3}{4}$

         $n = -5$

8.   (3, -5), (s, 0) $m = -\frac{5}{3}$

         $s = 0$

9.   (2, r), (4, ¼) $m = -\frac{1}{4}$

         $r = \frac{3}{4}$

10.   (1, 2), (7, t) $m = \frac{7}{6}$

         $t = 9$

Find the slope of the line. (Answer Key Continued)

11.
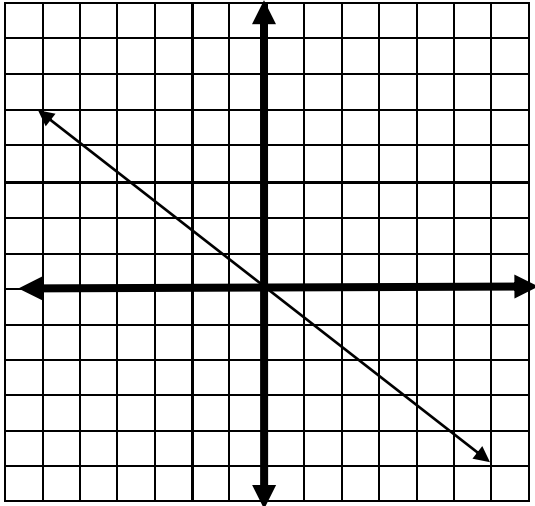
$$m = -\frac{9}{11}$$

14.
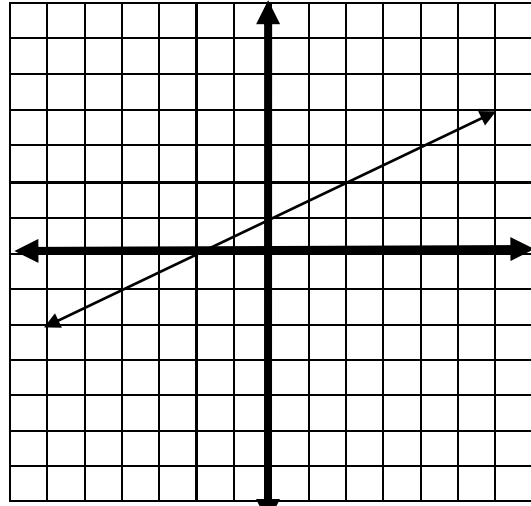
$$m = \frac{1}{2}$$
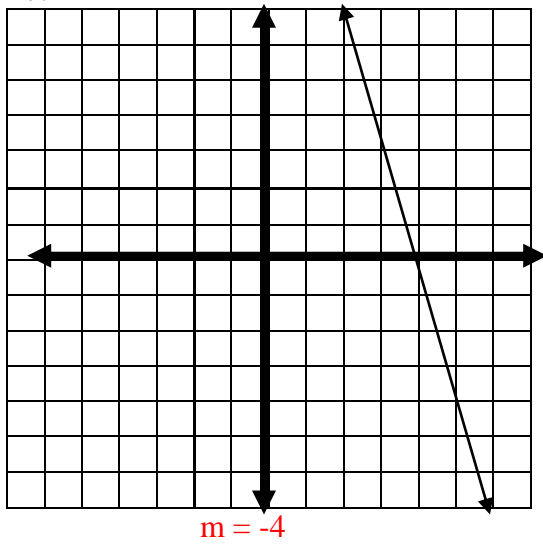
12.
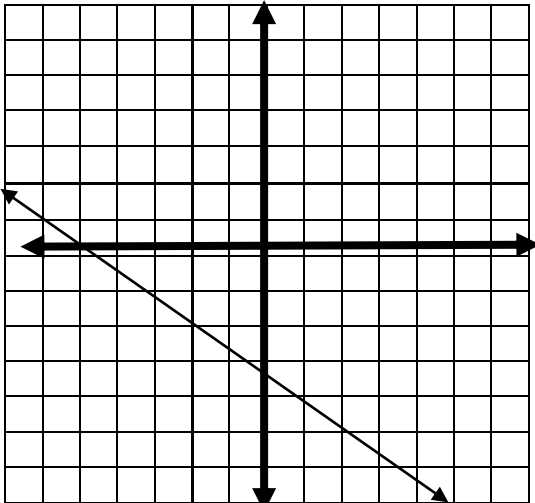
$$m = -\frac{3}{4}$$

15.
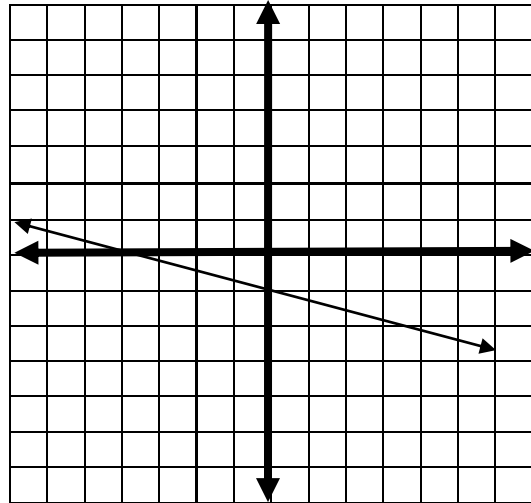
$$m = -\frac{1}{4}$$

13.
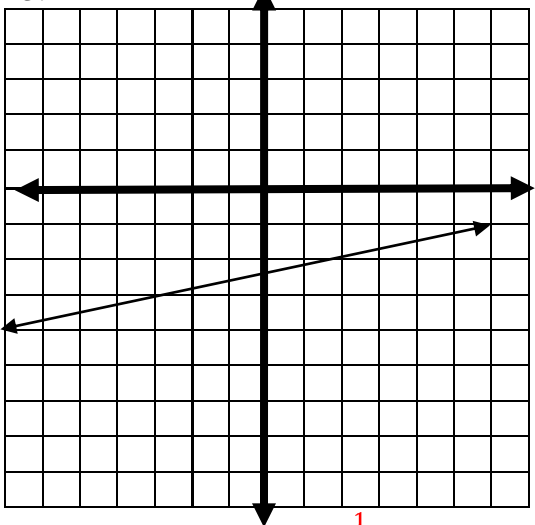
$$m = \frac{1}{4}$$

16.
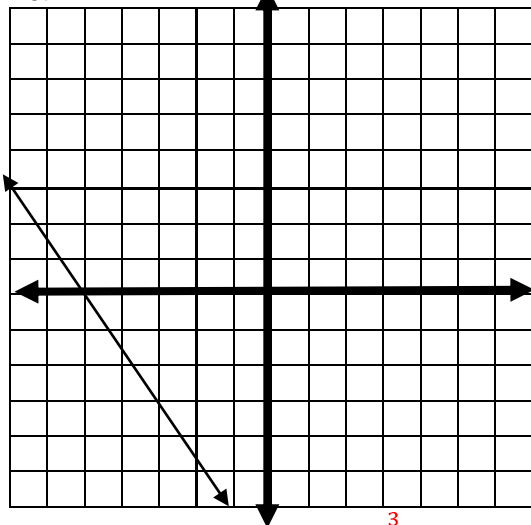
$$m = -\frac{3}{2}$$

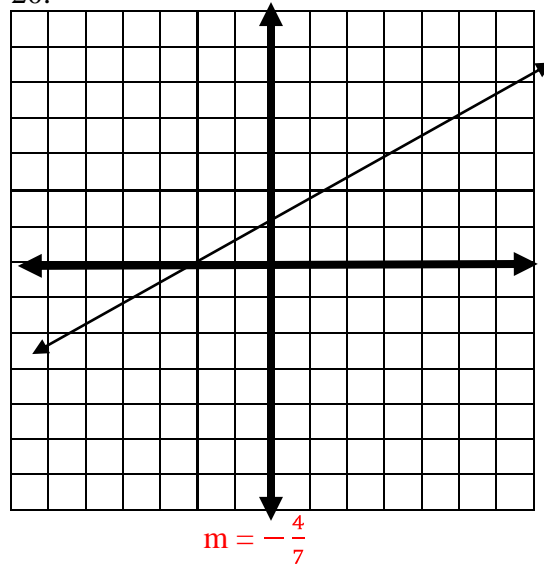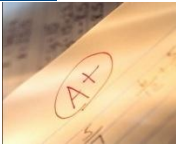(Answer Key Continued)

17.

$m = -4$

19.

$m = \frac{9}{4}$

18.

$m = \text{undefined}$

20.

$m = -\frac{4}{7}$
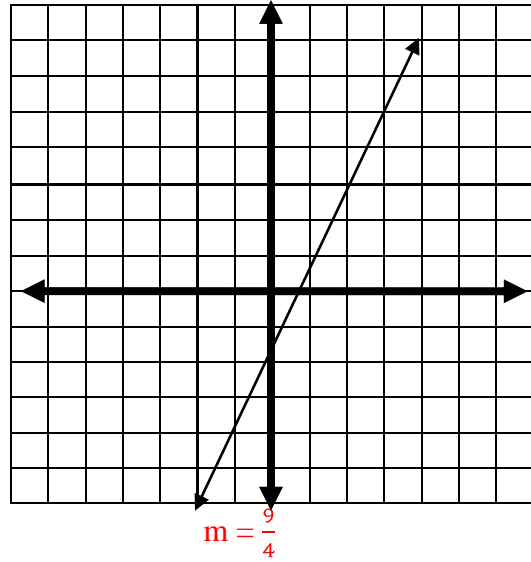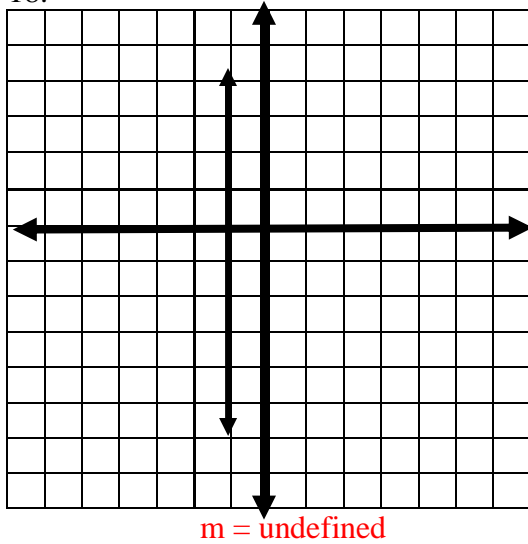
Slopes
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Student Practice Worksheet
Rubric

Criteria					
	4	3	2	1	0
Mechanics	No math errors	No major math errors or serious flaws in reasoning.	May be some serious math error or flaws in reasoning.	Major math errors or serious flaws in reasoning.	Blank answers

Quiz Grading Rubric:

Problem	Total points of Correct Answer		Problem	Total points of Correct Answer	
1.			11.		
2.			12.		
3.			13.		
4.			14.		
5.			15.		
6.			16.		
7.			17.		
8.			18.		
9.			19.		
10.			20.		

Slopes
~
Quiz

Name_____Date_____Grade_____

Find the slope of the line through each pair of points. Show all work. Pencil only. No Calculators.

1.   (1, -19), (-2, -7)   2.   (6, -10), (-15, 15)

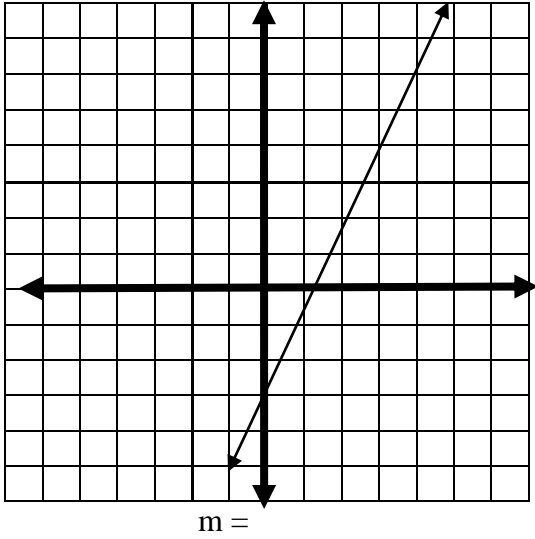3.   (3, 0), (-11, -15)   4.   (r, -12), (15, -3) m = 1
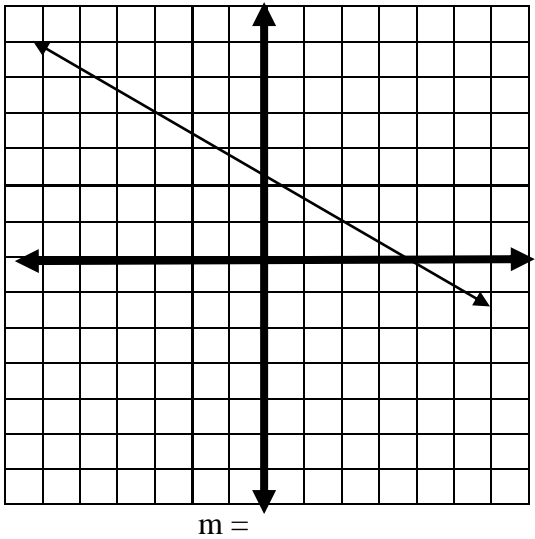
5.   (3, -20), (5, t) m = 14

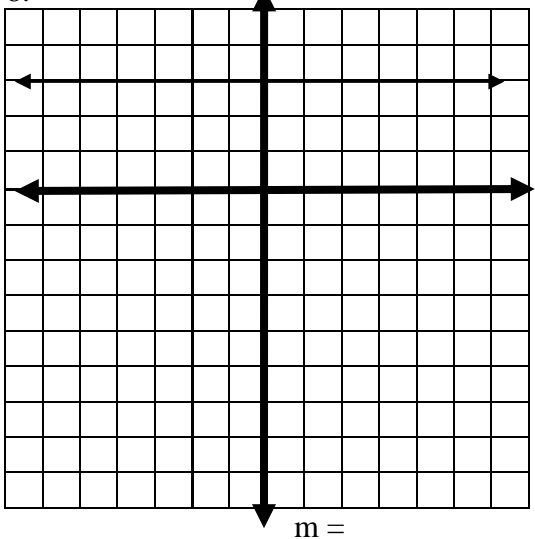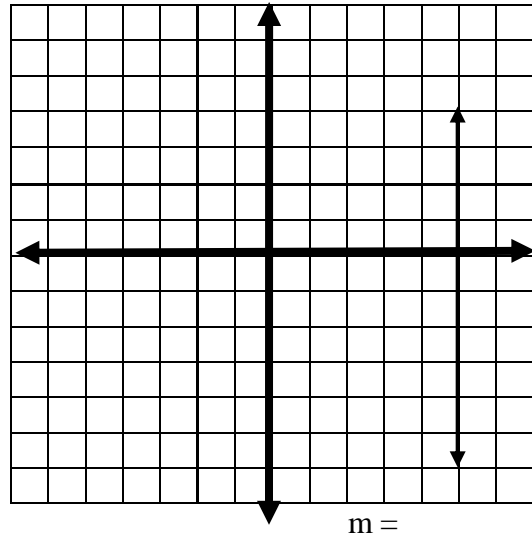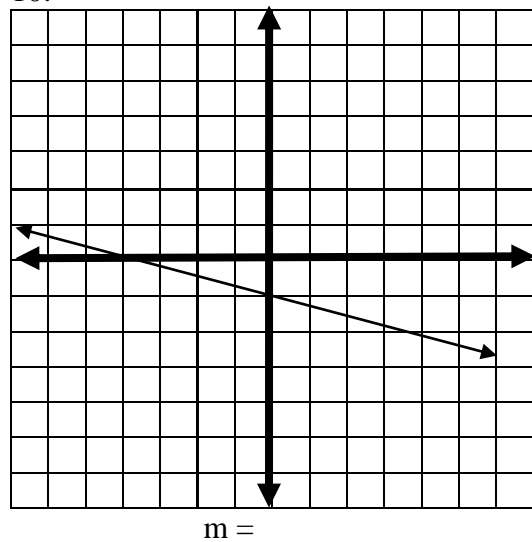Find the slope of each line. (Quiz continued)

6.

m =

7.

m =

8.

m =

9.

m =

10.

m =

Slopes
~
Quiz
Answer Key

Name_____Date_____Grade_____

Find the slope of the line through each pair of points. Show all work. Pencil only. No Calculators.

1.   (1, -19), (-2, -7)

$m = 4$

2.   (6, -10), (-15, 15)

$m = -\dfrac{25}{21}$

3.   (3, 0), (-11, -15)

$m = \dfrac{15}{14}$
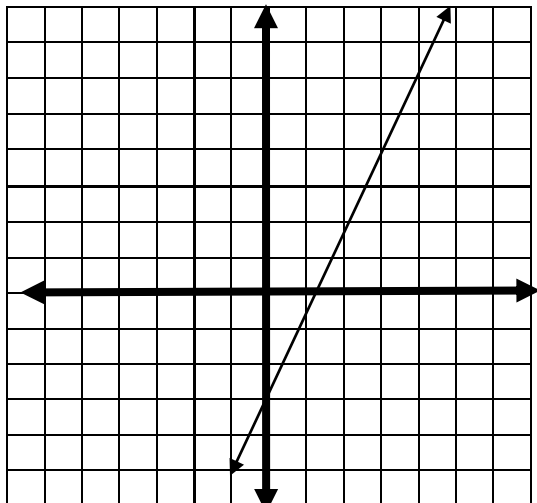
4.   (r, -12), (15, -3) m = 1

$r = 6$

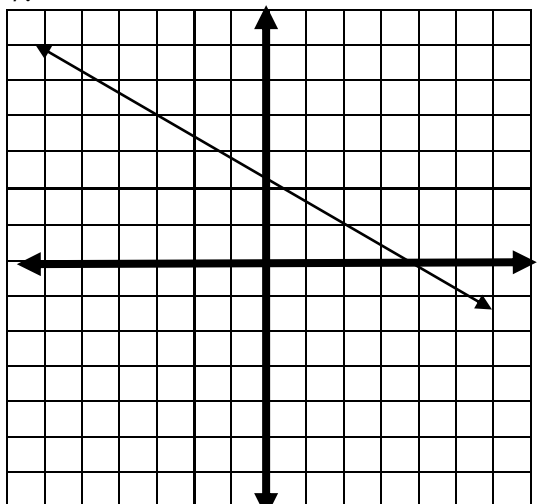5.   (3, -20), (5, t) m = 14

$t = 8$

Find the slope of each line. (Quiz continued) (Answer Key Continued)
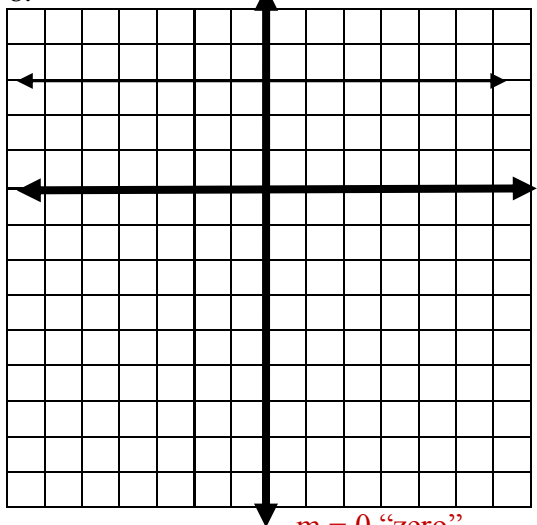
6.

$$m = \frac{3}{4}$$

7.

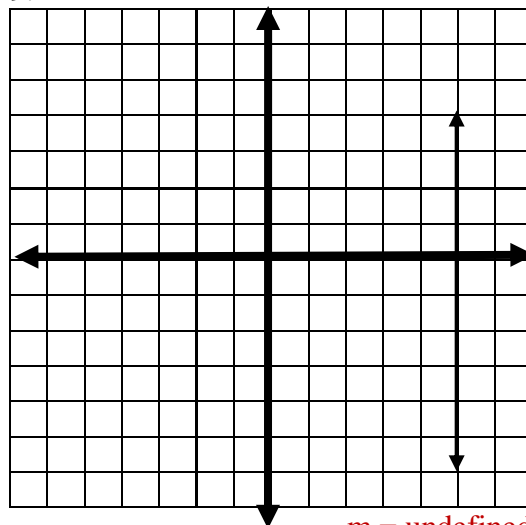$$m = -\frac{3}{5}$$

8.

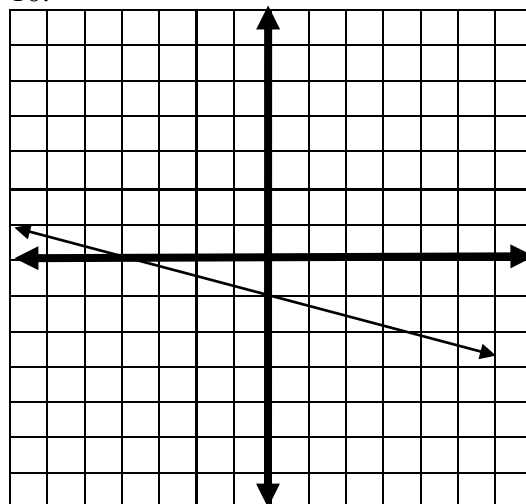$$m = 0 \text{ "zero"}$$

9.

$$m = \text{undefined}$$

10.

$$m = -\frac{1}{4}$$

Slopes

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Quiz
Rubric

Criteria					
	4	3	2	1	0
Mechanics	No math errors	No major math errors or serious flaws in reasoning.	May be some serious math error or flaws in reasoning.	Major math errors or serious flaws in reasoning.	Blank answers

Quiz Grading Rubric:

Problem	Total points of Correct Answer		Problem	Total points of Correct Answer	
1.			6.		
2.			7.		
3.			8.		
4.			9.		
5.			10.		