Rational Numbers
Comparing Rational Numbers
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Lesson Plan

I. Topic: Comparing Rational Numbers

II. Goals and Objectives:
A. The students will demonstrate an understanding of rational numbers.
B. The students will learn about negative and positive numbers.
C. The students will learn to use the <, >, and = signs.
D. The students will distinguish between opposite numbers.

III. Southern Union Mathematics Standards:

1. CM.2.1
Concepts (number sense, algebraic and geometric thinking, measurement, data analysis and probability)

2. CM.2.2
Problem-solving skills (explore, plan, solve, verify.)

3. PA.3.3
Perform calculations with and without technology in life situations.

4. PA.4.2
Identify numbers and relationship among numbers.

5. PA.7.1
Find and interpret information from graphs, charts, and numerical data.

IV. Materials:
A. Whiteboard with dry-erase markers (Blackboard with chalk could also be used.)
B. Ruler
C. Pencils
D. Comparing Integers Worksheets (Practice Worksheet, Quiz Worksheet).

V. Presentation Outline:
A. Introduction "Comparing Rational Numbers”

B. Key Concepts

C. Understanding less than (<), greater than (>), and equal (=) numbers
   I. Graph
   II. Examples

D. Ordering integers from least to greatest and greatest to least.
   I. Graph
   II. Examples

E. Graphing on the number line.
   Examples

F. Rational numbers and variables.
   Examples

VI. Presentation:
   A. Presentation Notes
   B. Power Point Presentation

VII. Independent Practice: Comparing Rational Numbers Worksheet
   A. Class work: # 2 - 50 Evens

   B. Homework: #1 - 49 Odds

   C. Due 2 days from the day assigned. Allow students to complete those questions which they did not complete in class.

VIII. Topic Assessment: Comparing Rational Numbers Quiz
   A. Answer questions from homework.

   B. 25-Question Quiz: 20 – 25 minutes

Rational Numbers

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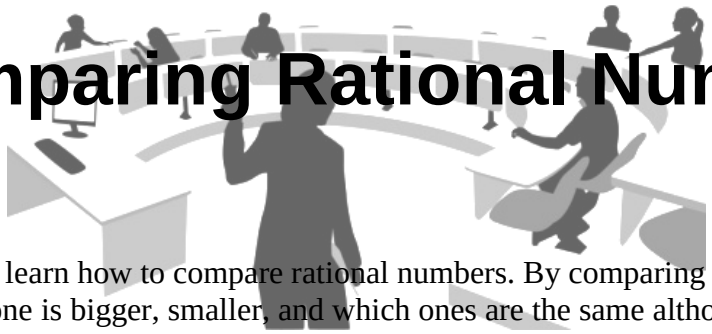
Comparing Rational Numbers



Introduction

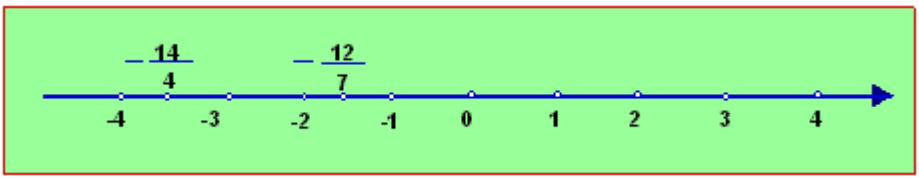
It is important that we learn how to compare rational numbers. By comparing rational numbers, we are able to decide which one is bigger, smaller, and which ones are the same although they might look different. We can also see where the rational numbers belong on the number line. A good way to compare two rational numbers is graphing them on a number line. On a number line, the rational number to the right of another rational number is greater.

Let us compare $\frac{-14}{4}$ and $\frac{12}{7}$.

By placing these two rational numbers on the number line, we can with ease compare the two.

Graph these points on a number line. To find where they go on the number line, it is easier to look at them as mixed numbers than as improper fractions. Another way to place them accurately on the number line is to make them into decimals.

Using this method, $-\frac{14}{4} = -3.5$ and $-\frac{12}{7} = -1.71\ldots$



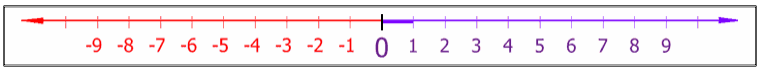
As you see, $-\frac{12}{7}$ is to the right of $\frac{-14}{4}$. So, $-\frac{12}{7} > \frac{-14}{4}$.

Key Concepts.

Some of the important concepts and words used in this section are the following:

- Rational Numbers
  Rational numbers are numbers which can be expressed as the quotient of two integers: such as *a/b*, where b is not equal to zero.

- Number Line
  A number line is a picture of a straight line on which every point is assumed to correspond to a real number



- Greater than (>)
  The notation $a > b$ means that a is greater than b.

- Less than (<)
  The notation $a < b$ means that a is less than b.

- Equal (=)
  The notation $a = b$ means that a has the same value as b.

- Variables
  A variable is a symbol that stands for a value that may vary; the term usually occurs in opposition to constant, which is a symbol for a non-varying value.

Understanding less than (<), greater than (>), and equal (=) numbers.

We use the following signs to compare different rational numbers without having to state their differences.

Less than (<): When one rational number is smaller than another rational number, we say that the number is "less than" the number. The sign we use to show it is less than the number is "<."

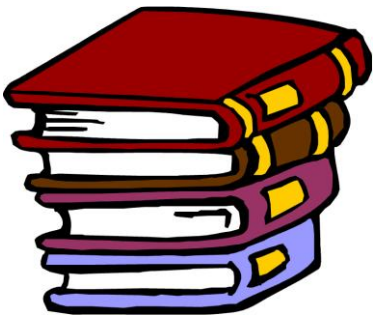
For example: a < b, 3 < 10, -2 < 0, 1/3 < 1/2.

Greater than (>): When one rational number is larger or has greater value than another rational number, then we say that the number is "greater than" the number. The sign we use to show that the number is greater than the number is ">."

For example: b > a, 10 >3, 0 > -2, 1/2 > 1/3.

Equal (=): One of the most common signs in mathematics is the equal sign "=." It is used to compare two different rational numbers, equations, symbols, etc. When two rational numbers are the same, or the left side of an equation equals the right side of the equation, we use the equal (=) signs.

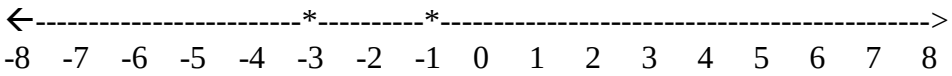
For example: a = b, -2 = -2, 75/3 = 25.



Example I

To compare integers, plot the points on the number line. The number farther to the right is the larger number.

Compare 1 and -3:

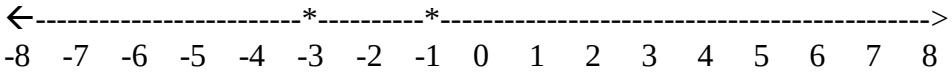


Since 1 is to the right of -3, $1 > -3$; or since -3 is to the left of 1, $-3 < 1$.

Example II

To compare integers, plot the points on the number line. The number farther to the right is the larger number.

Compare 1 and -3:

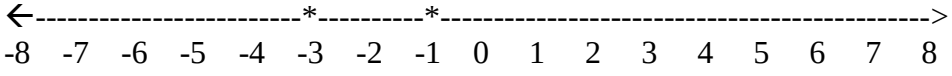


Since 1 is to the right of -3, $1 > -3$; or since -3 is to the left of 1, $-3 < 1$.

Example III

To compare integers, plot the points on the number line. The number farther to the right is the larger number.

Compare 1 and -3:



Since 1 is to the right of -3, $1 > -3$; or since -3 is to the left of 1, $-3 < 1$.

Ordering integers from least to greatest and greatest to least.

To understand ordering least to greatest, we must understand the words:

- Least
- Greatest

Once we understand these words, we are ready to understand Ordering Least to Greatest.

So, let's look at the words and learn their meaning:

Least = the smallest.

In the following list of numbers {6,12,3,15,9}, 3 is the smallest. 3 is the least.

Greatest = the largest.

In the following list of numbers {6,3, 9,15, 12}, 15 is the largest. 15 is the greatest.

Example 1

List the following numbers from least to greatest. When you do, every number on the left needs to be smaller than the number on its right.

a. {10, 4, -2, 11, 0, -4, 3, 7, 1} Answer → { -4, -2, 0, 1, 3, 4, 7, 10, 11}

Always make sure that the number to the left is smaller than the number to the right. Let's try it again.

b. {0.916, 0.18, 0.75, 0.321, 0.9} Answer → {0.18, 0.321, 0.75, 0.9, 0.916}

c. $\left\{0.25, \frac{17}{54}, \frac{5}{27}, \frac{44}{63}, \frac{13}{32}, 0.625\right\}$ Answer → $\left\{\frac{5}{27}, 0.25, \frac{17}{54}, \frac{13}{32}, 0.625, \frac{44}{63}\right\}$

Remember:

You can ALWAYS use the number line to help you determine which number is larger and which one is smaller. You can begin by placing all numbers on the number line first. The number which is farthest to the left is the smallest number. The number which is farthest to the right is the largest number.

Example 2

List the following numbers from greatest to least. When you do, every number on the left needs to be larger than the number on its right.

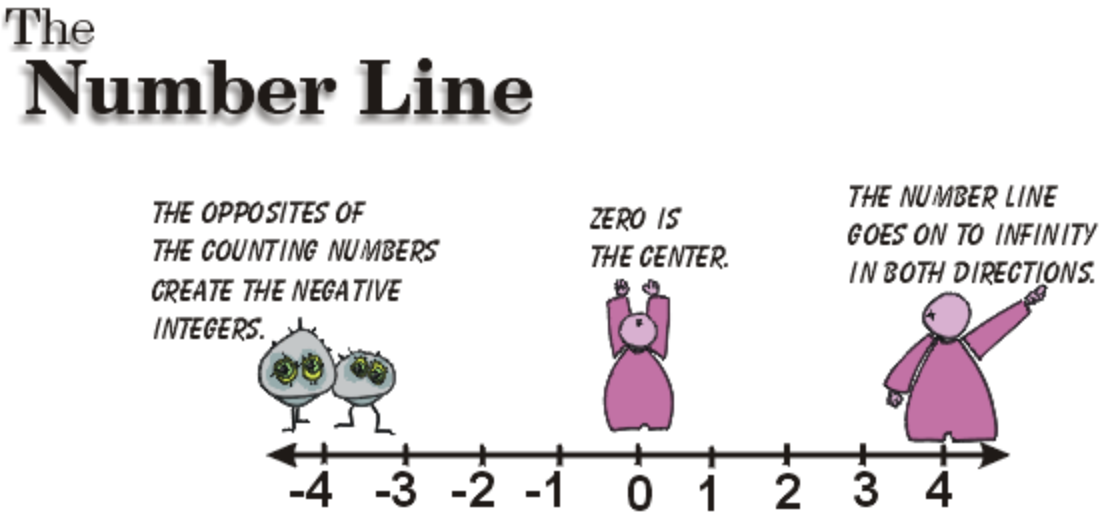
a. {-8, 3, -2, -74, 100, -101, 10, -26, 0, 17} Answer → {100, 17, 10, 3, 0, -2, -8, -26, -74, -101}

Always make sure that the number to the left is larger than the number to the right. Let's try it again.

b. {0.1428, 0.06, 0.9404, 0.4, 0.975, 0.625} Answer → {0.975, 0.625, 0.9404, 0.4, 0.1428, 0.06}

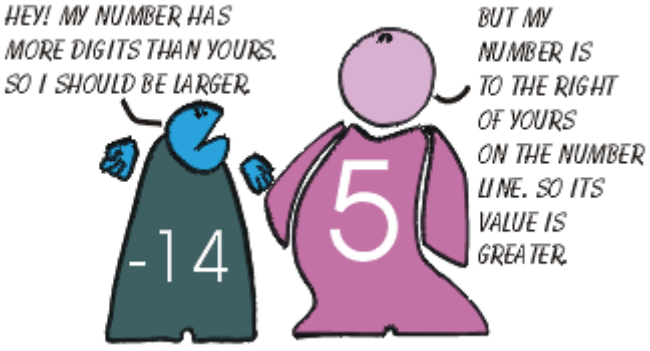
c. $\left\{\frac{7}{20}, \frac{19}{28}, \frac{25}{26}, \frac{21}{25}, 0.6, 0.1, 0.875\right\}$ Answer → $\left\{\frac{25}{26}, 0.875, \frac{21}{25}, \frac{19}{28}, 0.6, \frac{7}{20}, 0.1\right\}$

Graphing on the number line.



Comparing and Ordering Integers

- The set of integers is composed of the counting (natural) numbers, their opposites, and zero.
- Beginning with zero, numbers, increase in value to the right (0, 1, 2, 3, …) and decrease in value to the left (…-3, -2, -1, 0).
  When comparing numbers the order in which they are placed on the number line will determine if each is greater than or less than another number.



If a number is to the left of a number on the number line, it is less than the other number. If it is to the right, then it is greater than that number.

Example: If the lowest score wins, order the following golf scores from best to worst: Tigre Madera –4, Jack Nickles +1, Nick Cost –2, Freddy Pairs –5, John Weekly +3
Answer: -5, -4, -2, +1, +3

http://www.learningwave.com/chapters/integers/numline.html

Rational numbers and variables.

Variables

A variable is a letter that can represent a range of numbers, depending on its usage. The most common designations of variables are **x** and **y**, since they also represent the axes on a graph. However, don't be surprised to see any of the letters from the alphabet: **n, m, a, b, c, t, r**, and **s** are often used.

Adding and Subtracting Like Terms

There is a very simple property for adding and subtracting algebraic expressions. To be able to add or subtract expressions, we must have *like terms.*

***Like terms* are terms that contain the same variable or group of variables raised to the same exponent, regardless of their numerical coefficient.**

For example:

- $3x$ and $6x$ are *like terms*. They both contain x.
- $6c^2$ and $19c^2$ are *like terms*. They both contain c^2.
- $2xy^3$ and $101xy^3$ are *like terms*. They both contain xy^3.
- km^2x^5 and $17km^2x^5$ are *like terms*. They both contain km^2x^5.

Notice that to determine *like terms*, you must consider the variables in each term as a group. *Like terms* are those with exactly the same variables raised to the same exponent. If two terms have the same variables, but to different powers, they are not *like terms* and cannot be combined. For example:
x^4 and $3x^2$ are not like terms since one contains x^4 and the other contains x^2 as variables.
$5vk^3$ and vk^2 are not like terms since one contains vk^3 and the other contains vk^2 as variables.

Adding and Subtracting Like Terms

To combine *like terms*, do the following:

1. Determine which terms contain the same variable or groups of variables raised to the same exponent.
2. Add or subtract the numerical coefficients.
3. Attach the common variables and exponents.

For example, $3x + 6x$ can be simplified to $(3 + 6)\,x = 9x$.

Example I

If possible, simplify each of the following expressions:

$$10a + 10b - 3a$$

To find the solution:

1. Determine which terms contain the same variable or groups of $10a + 10b - 3a$

variables raised to the same exponent: If we look at this equation we see that there are two terms which contain the variable *a*.

2. Add or subtract the numerical coefficients: We use the distributive property to rewrite the equation. Then we perform the subtraction indicated, subtracting 3 from 10. $(10 - 3)\,a + 10b$

3. Attach the common variables and exponents: We then display the final result from the subtraction. $7a + 10b$

Example II

If possible, simplify each of the following expressions:

$$5b^2 + 8b^3$$

To find the solution:

Determine which terms contain the same variable or groups of variables raised to the same exponent: While both terms have *b*'s in them, they are raised to different powers, b^2 and b^3. This means we cannot combine these two terms. $5b^2 + 8b^3$

Example III

If possible, simplify each of the following expressions:

$$3x^2y^2z - 5xyz + x^2y^2z$$

To find the solution:

1. Determine which terms contain the same variable or groups of variables raised to the same exponent: If we look at this equation we see that there are two terms which contain the variable x^2y^2z. $3x^2y^2z - 5xyz + x^2y^2z$

2. Add or subtract the numerical coefficients: We group these terms together and perform the addition indicated, adding 3 and 1 together. $(3 + 1)(x^2y^2z) - 5xyz$

3. Attach the common variables and exponents: Here we have the final result. $4x^2y^2z - 5xyz$

Combining like terms is crucial in solving equations. This is a procedure you will use often with algebraic expressions.

Comparing Integers
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Student Practice Worksheet

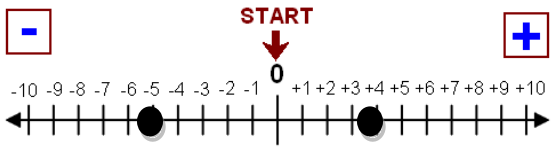
Name_____Date_____Grade_____

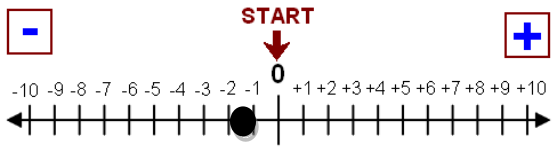
Answer the following questions regarding numbers and integers.

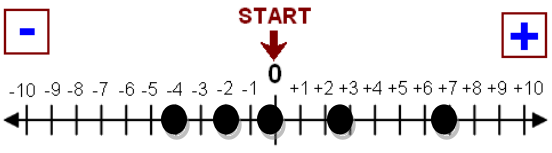
Compare the integers using <, >, or =.

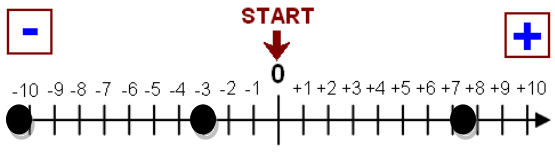
1.	-1 [ ] 0	2.	13 [ ] 0	3.	-4 [ ] 9
4.	-8 [ ] -10	5.	11 [ ] 13	6.	-11 [ ] -1
7.	-7 [ ] 3	8.	1 [ ] 13	9.	11 [ ] -9
10.	4 [ ] -2	11.	8 [ ] -8	12.	5 [ ] 11
13.	2 [ ] -3	14.	5 [ ] -12	15.	15 [ ] -5
16.	-5 [ ] 8	17.	9 [ ] -11	18.	-12 [ ] 6
19.	-12 [ ] -12	20.	-10 [ ] 13	21.	-2 [ ] -8

Name the set of numbers graphed.

22. 

23. 

24. 

25. 

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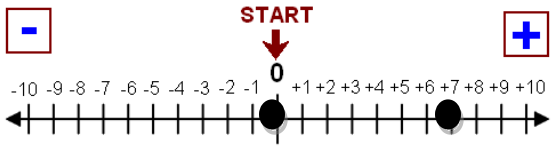
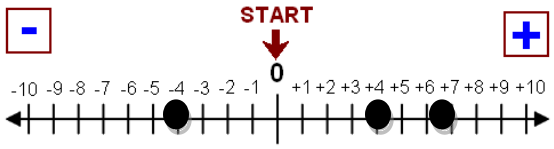
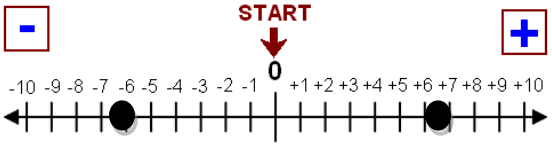
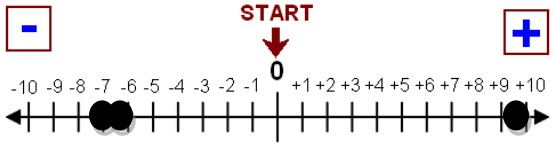
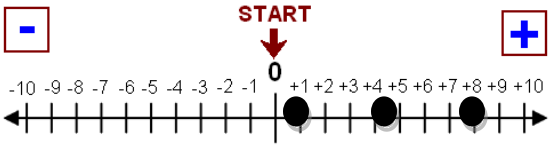
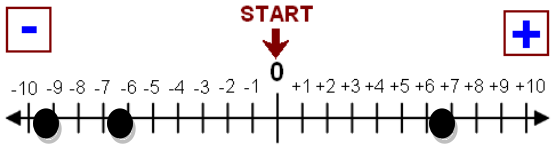
26. 

27. 

28. 

29. 

30. 

31. 

Draw a number line and plot each set of numbers:

32. {-4, -2, -1, 1, 3}

33. {0, 2, 5, 6, 9}

34. {Integers less than -7 or greater than -1}

35. {Integers greater than -5 and less than 9}

36. {-8.4, -7.2, -6.0, -4.8}

37. {-3, -0.5, 0.75, 2, 3}

(Student Worksheet Continued)

38. {-3, -1, 1, 3}

39. {-6, -2, 0, 2, 4, 9, 10}

40. {-7, -6, -5, -3, -1, 0, 2}

41. {-8, -5.5, 4, -3.5, 3, 5, 8.5}

42. {Integers less than or equal to -4}

Answer the following questions using these numbers {-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8}:

43. Which are whole numbers?

44. Which are natural numbers?

45. Which is the smallest positive integer?

46. Which is the greatest negative integer?

47. Which integer is neither positive nor negative?

48. Give an example of two numbers that are opposite:

49. On the horizontal number line, which direction is negative?

50. Which numbers are irrational numbers? Why?

Comparing Integers

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Student Practice Worksheet

Answer Key

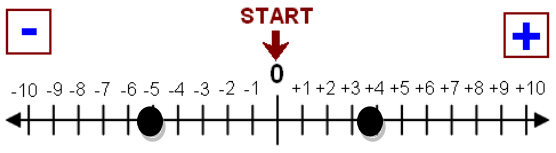
Name_____ Date_____ Grade_____

Answer the following questions regarding numbers and integers.

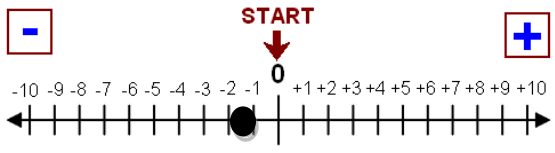
Compare the integers using <, >, or =.

1.	-1 [ ] 0	<	2.	13 [ ] 0	>	3.	-4 [ ] 9	<
4.	-8 [ ] -10	>	5.	11 [ ] 13	<	6.	-11 [ ] -1	<
7.	-7 [ ] 3	<	8.	1 [ ] 13	<	9.	11 [ ] -9	>
10.	4 [ ] -2	>	11.	8 [ ] -8	>	12.	5 [ ] 11	<
13.	2 [ ] -3	>	14.	5 [ ] -12	>	15.	15 [ ] -5	>
16.	-5 [ ] 8	<	17.	9 [ ] -11	>	18.	-12 [ ] 6	<
19.	-12 [ ] -12	=	20.	-10 [ ] 13	<	21.	-2 [ ] -8	>

Name the set of numbers graphed.

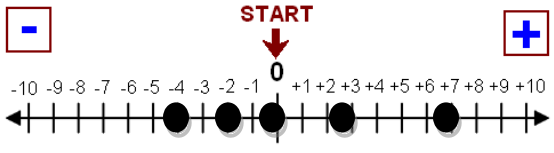
22. 

22. {-5, 4}

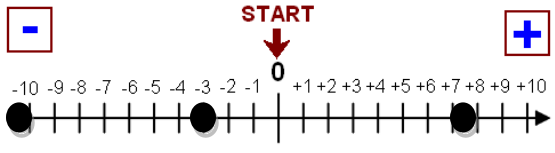
23. 

23. {-1}

24. {-4, -2, 0, 3, 7}

24. 

25. {-10, -3, 8}

25. 

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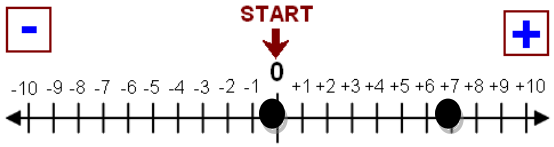
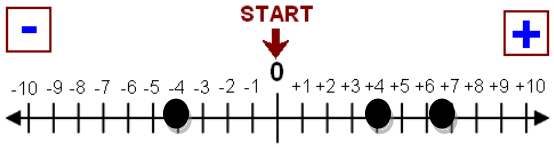
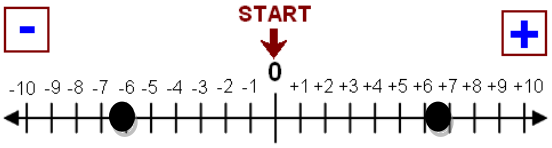
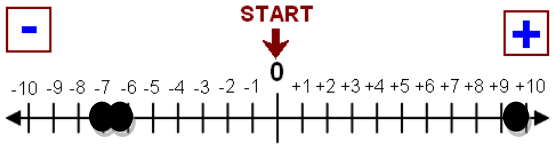
26. 

27. 

26. {0, 7}

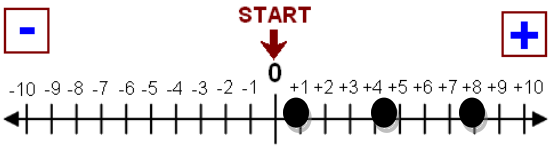
27. {-4, 4, 7}

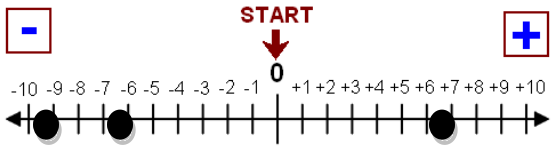
28. 

29. 

28. {-6, 7}

29. {-7, -6, 10}

30. 

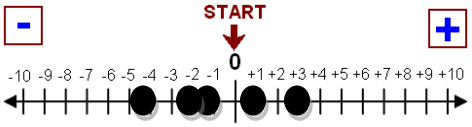
31. 

30. {1, 5, 8}

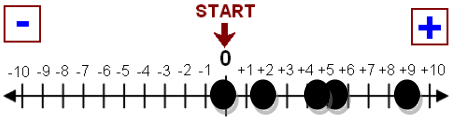
31. {9, 6, 7}

Draw a number line and plot each set of numbers.

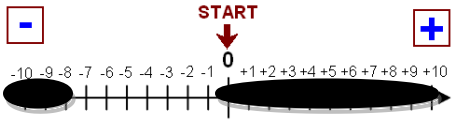
32. {-4, -2, -1, 1, 3}



33. {0, 2, 5, 6, 9}



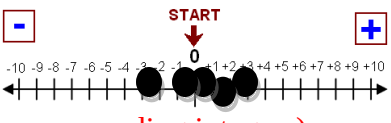
34. {Integers less than -7 or greater than -1}



35. {Integers greater than -5 and less than 9}

36. {-8.4, -7.2, -6.0, -4.8}

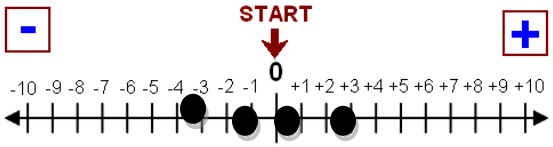
37. {-3, -0.5, 0.75, 2, 3}



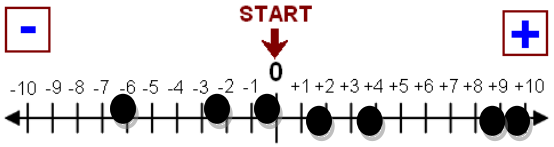
(36 and 37, and 41 have the dots between the corresponding integers)

(Student Worksheet Continued – Answer Key)

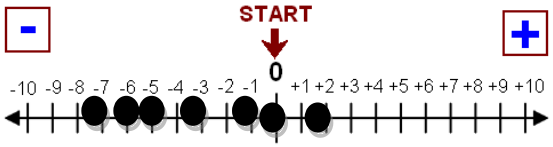
38. {-3, -1, 1, 3}



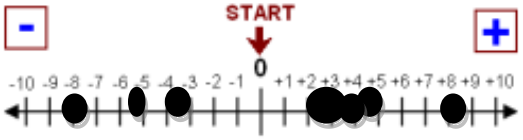
39. {-6, -2, 0, 2, 4, 9, 10}



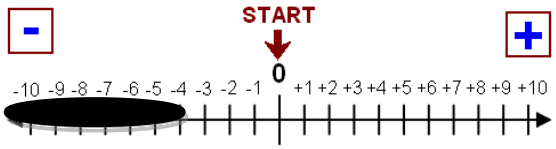
40. {-7, -6, -5, -3, -1, 0, 2}



41. {-8, -5.5, 4, -3.5, 3, 5, 8.5}



42. {Integers less than or equal to -4}



Answer the following questions using these numbers {-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8}

43. Which are whole numbers?
{0, 1, 2, 3, 4, 5, 6, 7, 8}

44. Which are natural numbers?
{1, 2, 3, 4, 5, 6, 7, 8}

45. Which is the smallest positive integer?
{1}

46. Which is the greatest negative integer?
{-1}

47. Which integer is neither positive nor negative?
{0}

48. Give an example of two numbers that are opposite:
Example: {-6, 6}

49. On the horizontal number line, which direction is negative?
Left

50. Which numbers are irrational numbers? Why? None.

Comparing Integers
~
Student Practice Worksheet
Rubric

Criteria					
	4	3	2	1	0
Mechanics	No math errors	No major math errors or serious flaws in reasoning.	May be some serious math error or flaws in reasoning.	Major math errors or serious flaws in reasoning.	Blank answers

Quiz Grading Rubric:

Problem	Total points of Correct Answer		Problem	Total points of Correct Answer	
1.			26		
2.			27		
3.			28		
4.			29		
5.			30		
6.			31		
7.			32		
8.			33		
9.			34		
10.			35		
11.			36		
12.			37		
13.			38		
14.			39		
15.			40		
16.			41		
17.			42		
18.			43		
19.			44		
20.			45		
21.			46		
22.			47		
23.			48		
24.			49		
25.			50		

Comparing Integers

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Quiz

Name_____Date_____Grade_____

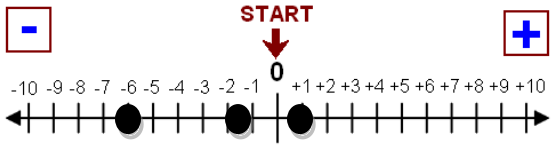
Answer the following questions regarding numbers and integers:

Compare the integers using <, >, or =.

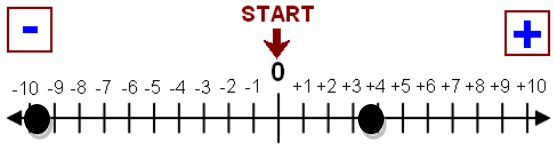
1. 9 [ ] 12
2. -3 [ ] 14
3. 11 [ ] -12
4. 13 [ ] 5
5. -4 [ ] 10
6. -2 [ ] 0
7. 7 [ ] -11
8. -6 [ ] -10
9. 4 [ ] -12
10. 14 [ ] -15

Name the set of numbers graphed.

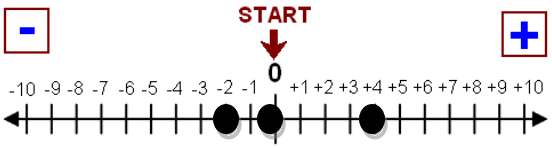
11.



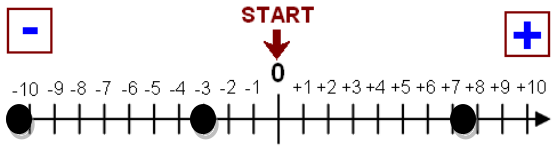
12.



13.



14.



(Quiz - Continued)

Draw a number line and plot each set of numbers.

15. {-6, -9, -5, 4, -1, -5}

16. {2, -6, -1, -2, 10, 2, 7, 8}

17. {3, 2, -7, 0, 6, 2, 5, -1, 4}

18. {1, 2, -3, 4, -5, 6, 7, 8, -9}

19. {4, -4}

20. {0}

True or False. Answer the following questions about rational numbers.

21. Are all whole numbers integers?

22. Are all natural numbers integers?

23. Are all whole numbers natural numbers?

24. Are all natural numbers whole numbers?

25. Are all whole numbers positive numbers?

Comparing Integers
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Quiz
Answer Key

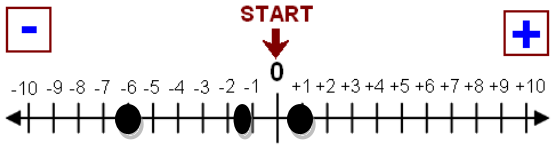
Name_____Date_____Grade_____

Answer the following questions regarding numbers and integers:

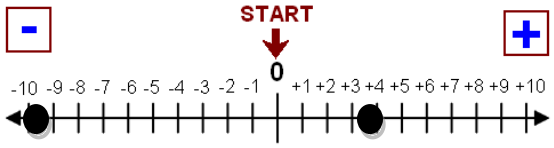
Compare the integers using <, >, or =.

1. 9 [ ] 12 <
2. -3 [ ] 14 <
3. 11 [ ] -12 >
4. 13 [ ] 5 >
5. -4 [ ] 10 <
6. -2 [ ] 0 <
7. 7 [ ] -11 >
8. -6 [ ] -10 >
9. 4 [ ] -12 >
10. 14 [ ] -15 >

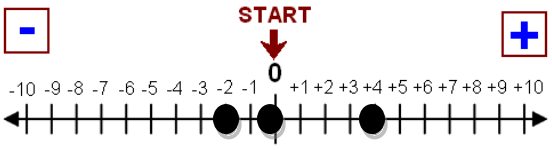
Name the set of numbers graphed.

11. 

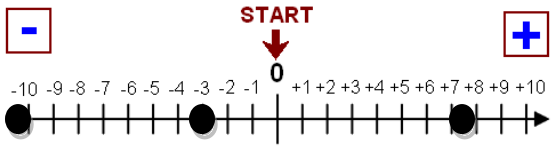
{-6, -1, 1}

12. 

{-10, 4}

13. 

{-2, 0, 4}

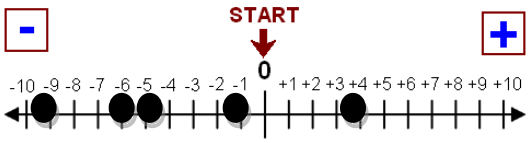
14. 

{-10, -3, 8}

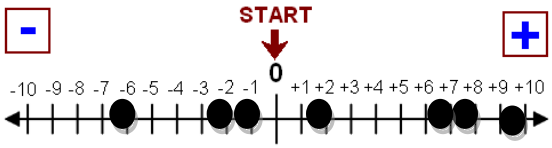
(Quiz – Continued – Answer Key)

Draw a number line and plot each set of numbers.

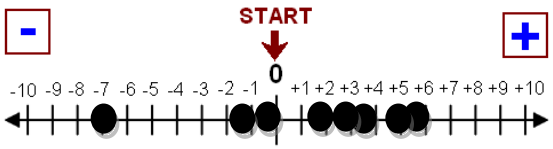
15. {-6, -9, -5, 4, -1, -5}



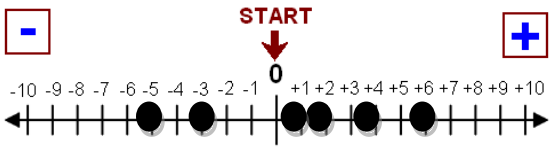
16. {2, -6, -1, -2, 10, 2, 7, 8}



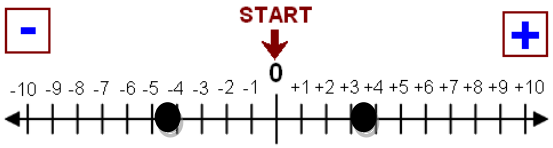
17. {3, 2, -7, 0, 6, 2, 5, -1, 4}



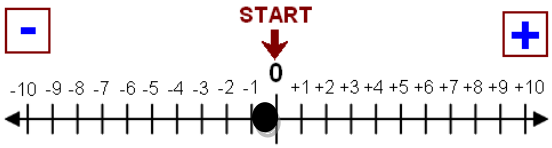
18. {1, 2, -3, 4, -5, 6, 7, 8, -9}



19. {4, -4}



20. {0}



True or False. Answer the following questions about rational numbers.

21. Are all whole numbers integers? True

22. Are all natural numbers integers? True

23. Are all whole numbers natural numbers? False

24. Are all natural numbers whole numbers? True

25. Are all whole numbers positive numbers? False

Comparing Integers

~

Quiz
Rubric

Criteria					
	4	3	2	1	0
Mechanics	No math errors	No major math errors or serious flaws in reasoning	May be some serious math error or flaws in reasoning	Major math errors or serious flaws in reasoning	Blank answers

Quiz Grading Rubric:

Problem	Total points of Correct Answer		Problem	Total points of Correct Answer	
1.			14.		
2.			15.		
3.			16.		
4.			17.		
5.			18.		
6.			19.		
7.			20.		
8.			21.		
9.			22.		
10			23.		
11.			24.		
12.			25.		
13.					